FORCE DISPLACEMENT RELATIONSHIP FOR THE PREDICTION OF STRUCTURAL FLUTTERING

by

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ABSTRACT

A new presentation of force displacement relationship for the prediction of structural fluttering is discussed in this paper. This topic is limited to a detailed discussion of the derivation of the equations of motion for the rigid streamlined airfoil/flat plate system of three degrees of freedom, which yields expressions for aerodynamic resultant forces due to a steady air flow. The equations of motion presented herein are derived from the conventional classical complex presentation transformed into real matrix equations. This new presentation of the equations of motion constitutes the foundation for the latest state-of-the-art of predicting critical wind loads on the basis of model tests in the wind tunnel.

INTRODUCTION

Economic considerations in construction industry and an increased demand for ergonomic designs gradually led structural engineers into designing lighter and more slender structural systems. This design tendency, however, was not without consequencies. The collapse of the Tacoma Narrows Bridge in 1940 in Washington, USA, gave the first remarkable signal to structural engineers on disastrous effect of wind-induced vibrations of slender structures.

Real structures which prove to be sensitive to vibrations on account of wind loads acting within the anticipated design velocity spectrum include cable stayed bridges, aircraft hangars (e.g. [6]), flat roof panels, tension girders in suspended roof structures etc.

The mechanism of wind-induced vibrations of an elastically restrained structure under wind action can be described through the energy being absorbed from passing wind flow. A set of differential equations leads to the familiar eigenvalue problem. Based on its solution, one can predict a critical wind speed which defines the critical point at which sustained or divergent self exited oscillations start to occur.

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The effect of this type of instability is known as "flutter phenomenon". The motion is termed self-exited in the sense that the aerodynamic forces maintaining it are a function of the structural displacements and their derivatives. The classical flutter phenomenon analysis found its origin in the study of airwing vibrations of aircrafts [5]. It can be shown [2,3,4] that a streamlined airfoil can be modelled for both experimental and analytical purposes as a rigid flat plate system.

The objective of this paper is to provide a new presentation of the equations of motion based on the classical flat plate model of three degrees of freedom. The equations of motion in the concept presented here constitute the basis for electronically monitored determination of aerodynamic forces to be discussed in a new paper, presently under preparation. Due to the complexity of the subject, a solution of the equations of motion which leads to the prediction of critical wind speeds, is beyond the scope of this paper and will be discussed in another paper.

Governing Differential Equations

Aerodynamic Model - The phenomenon of airfoil flutter is best described with respect to a classical airfoil model involving three coupled degrees of freedom h, α and ß as illustrated in fig. 1 :

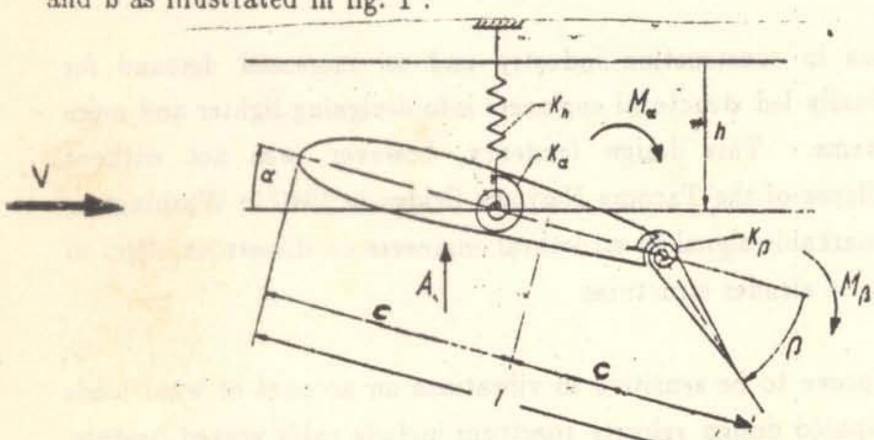

Fig. 1 : Elastically Restrained Airfoil with Three Degrees of Freedom

with k_h = flexural stiffness
k_α = torsional stiffness of the wing
k_β = torsional stiffness of the tail
c = reference geometrical length (e.g. half – chord length).
v = wind speed
A, M_α, M_β = aerodynamic forces
h, α, ß = coordinates

Since it was found that the airfoil subjected to wind action does exhibit a similar dynamic behaviour as a flat plate, the airfoil model of fig. 1, for structural analysis purposes, has been substituted with the flat plate model shown in fig. 2 below:

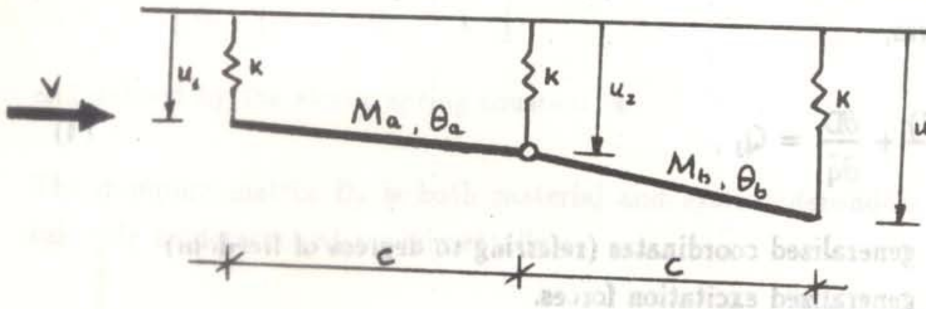

Fig. 2 : Elastically Restrained Flat-Plate Model

with M_a, M_b = flat plate masses
Θ_a, Θ_b = flat plate moment of inertia
k = elastic stiffness
c = reference geometrical length
v = wind speed
u_1, u_2, u_3 = coordinates

Careful inspection of Fig. 2 reveals that the model is elastically restrained on 3 vertical springs. The degrees of freedom of this system therefore still remains, 3, and as such is analogous to the model shown in fig. 1. The configuration of the system illustrated in fig. 2 has been chosen for experimental reasons not subject to discussion in this paper.

Equations of Motion - Equations of motion are derived on the basis of the Lagrange's principle of conservation of energy. The kinetic energy, E, of the vibrating flat plate system is given by

$$E = \frac{1}{2} M_a \left(\frac{\dot{u}_1+\dot{u}_2}{2}\right)^2 + \frac{1}{2} M_b \left(\frac{\dot{u}_2+\dot{u}_3}{2}\right)^2 + \frac{1}{2} \Theta_a \left(\frac{\dot{u}_2-\dot{u}_1}{c}\right)^2 + \frac{1}{2} \Theta_b \left(\frac{\dot{u}_3-\dot{u}_2}{c}\right)^2 \qquad (1)$$

Furthermore the potential energy takes the form

$$U = \frac{1}{2} k \left(u_1^2 + u_2^2 + u_3^2\right), \qquad (2)$$

whereas the dissipative energy, with d as damping mass, can be written to read :

$$D = \frac{1}{2} d \left(\dot{u}_1^2 + \dot{u}_2^2 + \dot{u}_3^2 \right). \tag{3}$$

Lagrange's equations of motion can be applied to describe the dynamic equilibrium of forces in the form,

$$\frac{\partial}{\partial t}\left(\frac{\partial E}{\partial \dot{q}_j}\right) - \frac{\partial (E-U)}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} = Q_j , \tag{4}$$

whereby q_j = generalized coordinates (referring to degrees of freedom)
Q_j = generalized excitation forces.

Making use of the expressions in eq. (1), (2) and (3) and substituting these into eq. (4), one is able to write the equations of motion in the compact form of eg. (5) below:

$$\underline{M}_s \, \underline{\ddot{U}} + \underline{D}_s \, \underline{\dot{U}} + \underline{K}_s \, \underline{U} = \underline{P} , \tag{5}$$

where $\underline{U}$ relates to the displacement and $\underline{P}$ to the force vector.
The crucial elements of eq. (5) are the mass matrix, $\underline{M}_s$, the damping matrix, $\underline{D}_s$, and the stiffness matrix, $\underline{K}_s$. The following can be said about these matrices:

The mass matrix, $\underline{M}_s$, assumes the form:

$$\underline{M}_s = \begin{bmatrix} \frac{M_a}{4} + \frac{\Theta_a}{c^2} & \frac{M_a}{4} - \frac{\Theta_a}{c^2} & 0 \\ \frac{M_a}{4} - \frac{\Theta_a}{c^2} & \frac{1}{4}(M_a + M_b) + \frac{1}{c^2}(\Theta_a + \Theta_b) & \frac{M_b}{4} - \frac{\Theta_b}{c^2} \\ 0 & \frac{M_b}{4} - \frac{\Theta_b}{c^2} & \frac{M_b}{4} + \frac{\Theta_b}{c^2} \end{bmatrix} \tag{6}$$

where the matrix elements M_{sij} $(1 \leq i, j \leq 3)$ are defined by the quantities shown in fig 2.

The stiffness matrix, $\underline{K}_s$, of the system with the elements $K_{ij} = k\delta_{ij}$ $(1 \leq i,j \leq 3)$ is diagonal in nature:

$$\underline{K}_s = \begin{bmatrix} k & | & 0 & | & 0 \\ - & - & - & - & - \\ 0 & | & k & | & 0 \\ - & - & - & - & - \\ 0 & | & 0 & | & k \end{bmatrix} \tag{7}$$

and defined by the elastic spring constant, k.

The damping matrix $\underline{D}_s$ is both material and system dependent and its elements can only be determined experimentally.

Finally, the displacement or response vector, $\underline{U}$, of the system takes the form

$$\underline{U} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \tag{8}$$

whereas, $\underline{P}$, the excitation force vector

$$\underline{P} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}. \tag{9}$$

For analytical reasons alone, it turns out to be convenient if one chooses to re-write the flat-plate equations of motion (eq. (5)) in terms of classical coordinates used in the airfoil model of fig.1. This leads to a form of the dynamic response vector:

$$\tilde{\underline{U}} = \begin{bmatrix} h \\ \alpha \\ \beta \end{bmatrix} \tag{10}$$

where the coordinates, h, relates to the translational and α, β to rotational response movements.

The transformation from the rectilinear vertical coordinates $\underline{U}$, to the generalized, classical, coordinates $\tilde{\underline{U}}$, can be achieved on the basis of the transformation matrix, $\underline{T}$, defined by the relationship,

$$\tilde{\underline{U}} = \underline{T}\,\underline{U}, \tag{11}$$

where,

$$\underline{T} = \begin{array}{c} \\ h \\ \alpha \\ \beta \end{array} \begin{array}{c} \begin{array}{ccc} u_1 & u_2 & u_3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{c} & \frac{1}{c} & 0 \\ \frac{1}{c} & -\frac{2}{c} & \frac{1}{c} \end{bmatrix} \end{array} \tag{12}$$

Hence, the equations of motion in terms of classical coordinates, $\underline{\tilde{U}}$, are obtained through an appropriate transformation of eq. (5). The equations of motion written in terms of generalized coordinates assume the following form,

$$\underline{\tilde{M}_s}\, \underline{\ddot{\tilde{U}}} + \underline{\tilde{D}_s}\, \underline{\dot{\tilde{U}}} + \underline{\tilde{K}_s}\, \underline{\tilde{U}} = \underline{\tilde{P}} \tag{13}$$

$$\text{with } \underline{\tilde{M}_s} = \underline{T}^{-T}\, \underline{M_s}\, \underline{T}^{-1}$$

$$\underline{\tilde{D}_s} = \underline{T}^{-T}\, \underline{D_s}\, \underline{T}^{-1} \tag{14}$$

$$\underline{\tilde{K}_s} = \underline{T}^{-T}\, \underline{K_s}\, \underline{T}^{-1}$$

Thus the mass matrix, $\underline{\tilde{M}_s}$, takes the form:

$$\underline{\tilde{M}_s} = \left[\begin{array}{c|c|c} M_a + M_b & \frac{c}{2}(M_a - M_b) & \frac{c}{2} M_b \\ \hline \frac{c}{2}(M_a - M_b) & (\Theta_a + \Theta_b) + (\frac{c}{2})^2 (M_a + M_b) & \Theta_b + (\frac{c}{2})^2 M_b \\ \hline \frac{c}{2} M_b & \Theta_b + (\frac{c}{2})^2 M_b & \Theta_b + (\frac{c}{2})^2 M_b \end{array}\right], \tag{15}$$

whereas the stiffness matrix, $\underline{\tilde{K}s}$, reads:

$$\tilde{\underline{K}}_s = \begin{bmatrix} 3k & 0 & ck \\ 0 & 2c^2k & c^2k \\ ck & c^2k & c^2k \end{bmatrix} \tag{16}$$

The reader should note the symmetry of both $\underline{M}_s$ and $\underline{K}_s$. In contrast to , $\underline{K}_s$, the transformed matrix $\tilde{\underline{K}}_s$ has become non-diagonal.

The excitation force, $\tilde{\underline{P}}$, is defined as

$$\tilde{\underline{P}} = \begin{bmatrix} -A \\ M_\alpha \\ M_\beta \end{bmatrix} \tag{17}$$

and its elements are displayed in fig. 3 below.

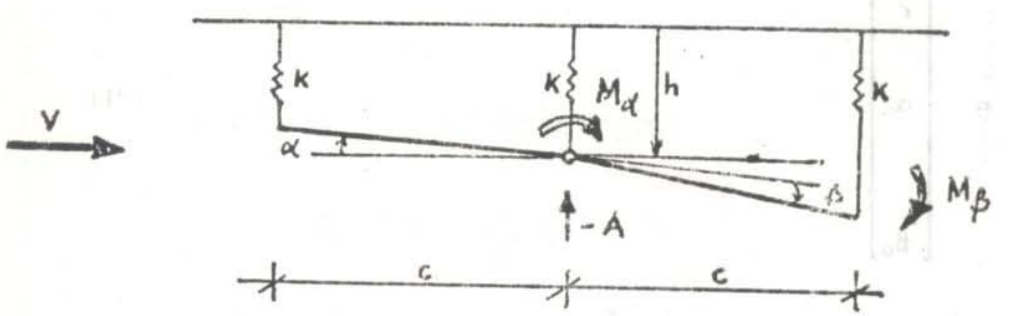

Fig 3: Illustration of Generalized Coordinates h, α, ß for flat-plate model.

Conventional Wind Load Matrix Coefficients

The work of Theodorsen [5] has shown that acting air force $\tilde{\underline{P}}$ is a function of the structural response $\tilde{\underline{U}}$. By assuming harmonic oscillations the response takes the following complex form:

$$\tilde{\underline{U}} = \tilde{\underline{U}}_o e^{i\omega t} , \tag{18}$$

for which $\tilde{\underline{U}}_o$ is the amplitude of oscillation and ω the frequency of the oscillating system.

For the sake of conveniency, the acting air force $\underline{\tilde{P}}$ is presented in a form containing dimensionless characters:

$$\underline{\tilde{P}} = \underline{F}_w \, \underline{\tilde{W}}^* \, \underline{\tilde{U}}_o^* \, e^{i\omega t} \, , \tag{19}$$

$$\underline{F}_w = \begin{bmatrix} L\pi\rho c^3\omega^2 & 0 & 0 \\ 0 & L\pi\rho c^4\omega^2 & 0 \\ 0 & 0 & L\pi\rho c^4\omega^2 \end{bmatrix} \tag{20}$$

In eq. (20) L represents the profile length whereas ρ the air density. In eq. (19) the displacement amplitude vector becomes dimensionless:

$$\underline{\tilde{U}}_o^* = \begin{bmatrix} \frac{h_o}{c} \\ \alpha_o \\ \beta_o \end{bmatrix} \tag{21}$$

and the complex wind load matrix $\underline{\tilde{W}}^*$ is dimensionless containing complex coefficients:

$$\underline{\tilde{W}}^* = \begin{bmatrix} k_a & k_b - \frac{1}{2}k_a & k_c \\ m_a - \frac{1}{2}k_a & m_b - \frac{1}{2}(k_b - m_a) + \frac{1}{4}k_a & m_c - \frac{1}{2}k_c \\ n_c & n_b - \frac{1}{2}n_a & n_c \end{bmatrix} , \tag{22}$$

The complex coefficients of the wind load in eq.(22) were derived by Theodorsen [5] as functions of the reduced frequency ω^*, with $\omega^* = \frac{\omega c}{v}$ (23)

whereby

ω = eigenfrequency of the structure
c = reference geometrical length
v = wind speed.

as follows:

$$k_a = 1 - i\frac{2}{\omega^*}C(\omega^*)$$

$$k_b = \frac{1}{2} - i\frac{1}{\omega^*}\{1 + 2C(\omega^*)\} - \frac{2}{\omega^{*2}}C(\omega^*)$$

$$k_c = \frac{2}{3\pi} - i\frac{1}{2\omega^*} - C(\omega^*)\{\frac{i}{\omega^*}(\frac{1}{2} + \frac{2}{\pi}) + \frac{1}{\omega^{*2}}(\frac{2}{\pi} + 1)\}$$

$$m_a = \frac{1}{2}$$

$$m_b = \frac{3}{8} - \frac{i}{\omega^*}$$

$$m_c = \frac{1}{16} + \frac{1}{3\pi} - i\frac{2}{3\pi\omega^*} - i\frac{1}{2\omega^*} - \frac{1}{\omega^{*2}\pi}$$

$$n_a = \frac{2}{3\pi} - i\frac{2}{\omega^*\pi}C(\omega^*) + i\frac{1}{2\omega^*}C(\omega)$$

$$n_b = \frac{1}{16} + \frac{1}{3\pi} - i\frac{1}{3\omega^*\pi} - i\frac{1}{4\omega^*} - i\frac{2}{\omega^*\pi}C(\omega^*) + i\frac{1}{2\pi}C(\omega^*)$$

$$-\frac{2}{\pi\omega^{*2}}C(\omega^*) + \frac{1}{2\omega^{*2}}C(\omega^*)$$

$$n_c = \frac{1}{32} + \frac{1}{2\pi^2} - i\frac{1}{8\omega^*} - i\frac{1}{2\pi\omega^*} - i\frac{(2 - \frac{\pi}{2})}{4\omega^*\pi}C(\omega^*) - i\frac{(2 - \frac{\pi}{2})}{\omega^*\pi^2}C(\omega^*)$$

$$+ \frac{1}{\omega^{*2}\pi^2} - \frac{1}{2\omega^{*2}\pi} - \frac{(2 - \frac{\pi}{2})}{\omega^{*2}\pi^2}C(\omega^*) - \frac{(2 - \frac{\pi}{2})}{2\omega^{*2}\pi}C(\omega^*) \quad (24)$$

In eq. (24) the following definition apply:

$i = \sqrt{-1}$ the imaginary unit.

$C(\omega^*)$ the Theodorsen function which is defined through the Henkel functions $H_j^{(2)}$ of the second order as:

$$C(\omega^*) = \frac{H_1^{(2)}(\omega^*)}{H_1^{(2)}(\omega^*) + i\ H_0^{(2)}(\omega^*)} = F(\omega^*) + i\ G(\omega^*) \tag{25}$$

with $F(\omega^*)$ = The real part of Theodorsen function.

$G(\omega^*)$ = The imaginary part of Theodorsen function.

Real Presentation of Wind Load Matrix Coefficients

The derivation of excitation force $\underline{\tilde{P}}$ as functions of the structural response and their derivatives with respect to time allows the presentation of the excitation force matrices to be symbolically associated with the system matrices as follows:

$$\underline{\tilde{M}}_s\ \underline{\ddot{\tilde{U}}} + \underline{\tilde{D}}_s\ \underline{\dot{\tilde{U}}} + \underline{\tilde{K}}_s\ \underline{\tilde{U}} = \underline{\tilde{M}}_w\ \underline{\ddot{\tilde{U}}} + \underline{\tilde{D}}_w\ \underline{\dot{\tilde{U}}} + \underline{\tilde{K}}_w\ \underline{\tilde{U}} \tag{26}$$

with $\underline{\tilde{M}}_w$ = wind "mass"

$\underline{\tilde{D}}_w$ = wind "damping"

$\underline{\tilde{K}}_w$ = wind "stiffness" .

The right side of equation (26) represents the aerodynamic (wind-induced) force and moment terms in N/m^2 and kNm/m^2, respectively.

Introducing an arbitrary, dimensional factor matrix, $\underline{F}_v$, such that

$$\underline{\tilde{M}}_s\ \underline{\ddot{\tilde{U}}} + \underline{\tilde{D}}_s\ \underline{\dot{\tilde{U}}} + \underline{\tilde{K}}_s\ \underline{\tilde{U}} = \underline{F}_v\left[\underline{\tilde{M}}_w^*\ \underline{\ddot{\tilde{U}}}^* + \underline{\tilde{D}}_w^*\ \underline{\dot{\tilde{U}}}^* + \underline{\tilde{K}}_w^*\ \underline{\tilde{U}}^*\right] \tag{27}$$

$$\text{with } \underline{F}_v = \begin{bmatrix} L\rho v^2 c & 0 & 0 \\ 0 & L\rho v^2 c^2 & 0 \\ 0 & 0 & L\rho v^2 c^2 \end{bmatrix}, \tag{28}$$

where L represents the length of profile and ρ the air density, leaves the right-hand side of eq. (26) with dimensionless coefficient matrices $\underline{\tilde{M}}_w^*$, $\underline{\tilde{D}}_w^*$ and $\underline{\tilde{K}}_w^*$ as well as dimensionless response vectors $\underline{\ddot{\tilde{U}}}^*$, $\underline{\dot{\tilde{U}}}^*$ and $\underline{\tilde{U}}^*$, eq. (27).

It can be shown that the dimensionless acceleration, velocity and displacement response vectors assume the form:

$$\underline{\ddot{\tilde{U}}}^* = \begin{bmatrix} \frac{c\ddot{h}}{v^2} \\ \frac{c^2\ddot{\alpha}}{v^2} \\ \frac{c^2\ddot{\beta}}{v^2} \end{bmatrix}, \qquad \underline{\dot{\tilde{U}}}^* = \begin{bmatrix} \frac{\dot{h}}{v} \\ \frac{c\dot{\alpha}}{v} \\ \frac{c\dot{\beta}}{v} \end{bmatrix}, \qquad \underline{\tilde{U}}^* = \begin{bmatrix} \frac{h}{c} \\ \alpha \\ \beta \end{bmatrix} \tag{29}$$

Upon introduction of the classical dimensionless parameter ω^* (reduced frequency), the dimensionless coefficient matrices of eq. (27), right-hand side, are given by the following form:

$$\underline{\tilde{M}}_w^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{M}_{w11}^* & \tilde{M}_{w12}^* & \tilde{M}_{w13}^* \\ \tilde{M}_{w21}^* & \tilde{M}_{w22}^* & \tilde{M}_{w23}^* \\ \tilde{M}_{w31}^* & \tilde{M}_{w32}^* & \tilde{M}_{w33}^* \end{bmatrix} \tag{30}$$

$$\underline{\tilde{D}}_w^* = \begin{bmatrix} \omega^* & 0 & 0 \\ 0 & \omega^* & 0 \\ 0 & 0 & \omega^* \end{bmatrix} \begin{bmatrix} \tilde{D}_{w11}^* & \tilde{D}_{w12}^* & \tilde{D}_{w13}^* \\ \tilde{D}_{w21}^* & \tilde{D}_{w22}^* & \tilde{D}_{w23}^* \\ \tilde{D}_{w31}^* & \tilde{D}_{w32}^* & \tilde{D}_{w33}^* \end{bmatrix} \tag{31}$$

$$\underline{\tilde{K}}_w^* = \begin{bmatrix} \omega^{*2} & 0 & 0 \\ 0 & \omega^{*2} & 0 \\ 0 & 0 & \omega^{*2} \end{bmatrix} \begin{bmatrix} \tilde{K}_{w11}^* & \tilde{K}_{w12}^* & \tilde{K}_{w13}^* \\ \tilde{K}_{w21}^* & \tilde{K}_{w22}^* & \tilde{K}_{w23}^* \\ \tilde{K}_{w31}^* & \tilde{K}_{w32}^* & \tilde{K}_{w33}^* \end{bmatrix} \tag{32}$$

The matrix coefficients in eq. (30), (31) and (32) contain a total number of 27 elements which can be proved to be real functions of the reduced frequencies, ω^*, given in eq. (23).

The 27 real functions are derived from the classical complex functions of Theodorsen as follows:

The wind "mass" coefficients assume a special case of constant values:

$$\tilde{M}^*_{w11} = -\pi$$

$$\tilde{M}^*_{w12} = 0$$

$$\tilde{M}^*_{w13} = -\frac{2}{3}$$

$$\tilde{M}^*_{w21} = 0$$

$$\tilde{M}^*_{w22} = -\frac{\pi}{8} \qquad (33)$$

$$\tilde{M}^*_{w23} = -\frac{\pi}{16}$$

$$\tilde{M}^*_{w31} = -\frac{2}{3}$$

$$\tilde{M}^*_{w32} = -\frac{\pi}{16}$$

$$\tilde{M}^*_{w33} = -\pi\left(\frac{1}{32}+\frac{1}{2\pi^2}\right)$$

The wind "damping" and wind "stiffness" coefficients depend on the complex Theodorsen function, $C(\omega^*)$, given in eq. (25).

The wind "damping" coefficients will read

$$\tilde{D}^*_{w11} = -\frac{2\pi F}{\omega^*}$$

$$\tilde{D}^*_{w12} = -\pi\left\{\frac{1}{\omega^*}+\frac{F}{\omega^*}+\frac{2G}{\omega^{*2}}\right\}$$

$$\tilde{D}^*_{w13} = -\pi\left\{\frac{1}{2\omega^*}+\frac{F}{2\omega^*}+\frac{2F}{\omega^*\pi}+\frac{2G}{\omega^{*2}\pi}+\frac{G}{\omega^{*2}}\right\}$$

$$\tilde{D}^*_{w21} = -\frac{\pi F}{\omega^*}$$

$$\tilde{D}^*_{w22} = -\pi\left\{\frac{1}{2\omega^*} - \frac{F}{2\omega^*} - \frac{G}{\omega^{*2}}\right\} \tag{33b}$$

$$\tilde{D}^*_{w23} = -\pi\left\{\frac{2}{3\pi\omega^*} + \frac{1}{4\omega^*} - \frac{F}{\pi\omega^*} - \frac{F}{4\omega^*} - \frac{G}{\pi\omega^{*2}} - \frac{G}{2\omega^{*2}}\right\}$$

$$\tilde{D}^*_{w31} = \pi\left\{\frac{F}{2\omega^*} - \frac{2F}{\pi\omega^*}\right\}$$

$$\tilde{D}^*_{w32} = \pi\left\{-\frac{2}{3\pi\omega^*} - \frac{1}{4\omega^*} - \frac{F}{\pi\omega^*} + \frac{F}{4\omega^*} - \frac{2G}{\pi\omega^{*2}} + \frac{G}{2\omega^{*2}}\right\}$$

$$\tilde{D}^*_{w33} = \pi\left\{-\frac{1}{8\omega^*} - \frac{1}{2\pi\omega^*} - \frac{(2-\frac{\pi}{2})F}{4\pi\omega^*} - \frac{(2-\frac{\pi}{2})F}{\pi^2\omega^*} - \frac{(2-\frac{\pi}{2})G}{\pi^2\omega^{*2}} - \frac{(2-\frac{\pi}{2})G}{2\pi\omega^{*2}}\right\}$$

whereas the wind "stiffness" coefficients take the form:

$$\tilde{K}^*_{w11} = \frac{2\pi G}{\omega^*}$$

$$\tilde{K}^*_{w12} = -\pi\left\{\frac{2F}{\omega^{*2}} - \frac{G}{\omega^*}\right\}$$

$$\tilde{K}^*_{w13} = -\pi\left\{\frac{2F}{\pi\omega^{*2}} + \frac{F}{\omega^{*2}} - \frac{G}{2\omega^*} - \frac{2G}{\pi\omega^*}\right\}$$

$$\tilde{K}^*_{w21} = -\frac{\pi G}{\omega^*}$$

$$\tilde{K}^*_{w22} = \pi\left\{\frac{F}{\omega^{*2}} - \frac{G}{2\omega^*}\right\} \tag{33c}$$

$$\tilde{K}^*_{w23} = -\pi\left\{\frac{1}{\pi\omega^{*2}} - \frac{F}{\pi\omega^{*2}} - \frac{F}{2\omega^{*2}} + \frac{G}{4\omega^*} + \frac{G}{\pi\omega^*}\right\}$$

$$\tilde{K}^*_{w31} = \pi\left\{\frac{2G}{\pi\omega^*} - \frac{G}{2\omega^*}\right\}$$

$$\tilde{K}^*_{w32} = \pi\left\{\frac{G}{\pi\omega^*} - \frac{G}{4\omega^*} - \frac{2F}{\pi\omega^{*2}} + \frac{F}{2\omega^{*2}}\right\}$$

$$\tilde{K}^*_{w33} = \pi\left\{\frac{1}{\pi^2\omega^{*2}} - \frac{1}{2\pi\omega^{*2}} - \frac{(2-\frac{\pi}{2})F}{\pi^2\omega^{*2}} - \frac{(2-\frac{\pi}{2})F}{2\pi\omega^{*2}} + \frac{(2-\frac{\pi}{2})G}{4\pi\omega^*} + \frac{(2-\frac{\pi}{2})G}{\pi^2\omega^*}\right\}$$

Conclusions

Based on the information presented in this paper, the following conclusions are set forth.

1. The explicit force–displacement relationship to predict the fluttering phenomenon observed on relative light weight, slender structure systems is presented to inform the engineer of a new analysis tool for assessing critical wind force effects.

2. The theoretical model under consideration is based on both theoretical and experimental evidence obtained from the study of airfoils and an idealized analogy of a flat–plate system [7,8].

3. With the exception of isolated cases, most elastically restrained structural systems under wind action exhibit a flutter phenomenon involving either two coupled or one degree of freedom. Both of the above response forms constitute special cases of the classical theory of coupled three degrees of freedom.

4. Therefore, on account of item 3, the equations of motion (Eq. (27)) as well as the aerodynamic coefficients of Eq. (30) – (32), specifically Eq. (33a) through Eq. (33c), have general validity. The actual development of specific information on wind load coefficients for structural objects will be discussed in another paper.

5. The dynamic force displacement relationship, presented in this paper, constitutes a major improvement in the assessment methodology of wind forces causing fluttering and thus would seem to be of great interest to the structural engineer in general.

List of Symbols

*	Notation for dimensionless parameter
c	Reference geometrical length
k	Spring constant
L	profile length
ω	eigenfrequency
F	real part of Theodorsen – function
G	imaginary part of Theodorsen – function
$M_{w_{ij}}, D_{w_{ij}}, K_{w_{ij}}$	wind load coefficients
$\underline{\dot{U}}$	Displacement vector in classical coordinates h, α, ß

$\underline{U}$	Displacement vector in vertical coordinates u_1, u_2, u_3
$\underline{T}$	Transformation matrix
$\underline{\tilde{M}}_s$, $\underline{\tilde{D}}_s$, $\underline{\tilde{K}}_s$	System matrices based on $\underline{\tilde{U}}$
$\underline{M}_s$, $\underline{D}_s$, $\underline{K}_s$	System matrices based an $\underline{U}$
$\underline{F}_v$	Dimensional factor matrix
$\underline{\tilde{M}}_w$, $\underline{\tilde{D}}_w$, $\underline{\tilde{K}}_w$	wind load matrices based on $\underline{\tilde{U}}$
$\underline{M}_w$, $\underline{D}_w$, $\underline{K}_w$	wind load matrices based on $\underline{U}$

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