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Review Manuscript

Review on State of Art of Smoothed Particle Hydrodynamics Method and its Advances in Solid Dynamics

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ABSTRACT

The world is witnessing continued collapse of both buildings and other structures during earthquakes, which is an example of a dynamic effect. One cause of such disasters may be inadequacies of techniques for modeling of these structures during analysis and design as compared to their actual responses during dynamic events. Also, techniques for numerical modeling and analysis of structures in place are meshed methods which do not accurately capture the actual behavior of structural elements especially under high dynamic actions due to the assumption that mesh geometry is unchanged geometry tend to change with respect to time step of such an action. In view of this, meshless techniques such as Smoothed Particles Hydrodynamics (SPH) prove to be promising. However, the application of SPH method especially in solid dynamics, still poses some challenges that reduce its efficiency and need respective improvements. This paper reviews advances so far done in SPH method and its application in solid dynamics with the key focus on weaknesses of the method and soundness of the recommended solutions through reviews from recent research, from which recommendations for further improvements have been presented as well. Findings from reviewed papers show that efforts towards improving various challenges on the classical SPH specifically on dynamics of solids have been done and are hereby acknowledged. However, critical areas that still pose attention and require further research include criticality on choice of most suitable kernel function that best fulfills all interpolant requirements, criteria for setting of smoothing length and general SPH formulation that appropriately represents dynamic problem of solids other than those which have been covered so far. Special attention on clear way of setting the initial and boundary conditions of the kernel domain is also needed.

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INTRODUCTION

Failures of various structures due to dynamic actions is still a global challenge as the world is witnessing continued collapses of various buildings and other structures during earthquakes, which is an example of a dynamic effect. One of the main causes of such structural failures is the inadequacies of techniques for modeling of these structures during analysis and design as compared to their actual responses during dynamic events (Asprone, *et. al*., 2008).

Most of the techniques for numerical

modeling and analysis of structures in place are meshed methods such as Finite Element Methods (FEMs) or Boundary Elements Methods (BEMs) and the recent ones are the Adaptive Finite Element Methods (AFEMs) which are categorized as *h* -refinement and its principle is that, the same mesh type is used with changing mesh size during time steps of an action, *p*-refinement where the order of the polynomial FE basis functions is increased during simulation with no changes in mesh geometry and r-refinement where the number of mesh nodes is always kept constant while allowing nodal positions to be concentrated to areas with higher gradients. (Zhao, Ho, & Fu, 2013). Now, the basic principle of these methods involves discretization of the structure into meshes and then the structure response is simulated from the responses of the discrete meshes at time steps of an action while assuming unchanged mesh geometry. (Liu & Liu, 2010) This procedure does not accurately capture the actual behavior of structural elements especially under high dynamic actions where large deformations in the structure occur and thus causing mesh distortion, which means, mesh geometry of the structure before dynamic event is quite different from the one after an event, i.e. mesh geometry of the discretized structure is subject to change at every time step of a dynamic action. Figure 1 and 2 below depict mesh geometry of a discretized beam element in undeformed and deformed states as the effect of the dynamic event. The scenario of keeping the same mesh geometry at every time step of a dynamic action, while it is not true, leads to inaccurate simulation of the structure during analysis, and hence a wrong design that may lead to structural collapse.

Figure 1: Undeformed meshed beam.

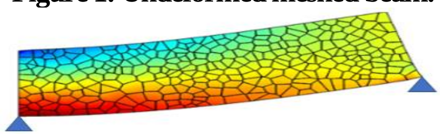

Figure 2: Mesh distortion in a deformed beam.

To ensure a structure subjected to dynamic action remains safe and continues to serve its intended purpose throughout the design period while fulfilling other requirements and guidelines as described in standards, manuals, research works, etc., they have to be analysed and designed accurately. This goal can be achieved if proper modelling and analysis techniques are employed. One of such techniques that seems to be more promising is the Smoothed particle hydrodynamics (SPH) method which belongs to the Lagrangian family. The principle of this method is that, a structural system is characterized by position coordinates of its components and time while associating mass, stiffness and damping properties of the system when it responds to dynamic actions. In SPH world, these structural components are assumed to be imaginary particles while position coordinates are known as nodes or supports (Price, 2010). Neither meshing nor node connectivity information are required for modelling a structure by SPH method, except defining particle position which is clearly defined by its coordinates and material properties. The global response of the structure under loads is then simulated as a vector sum of the responses of the individual particles forming the continuum (Monaghan, 1992).

However, the classical SPH method was not founded as the modelling and analysis method for solid dynamic structures, therefore the application of this method in this domain involved the series of developments of the classical one over decades, and in fact still further investigations on such improvements are inevitable to ensure satisfactory performance of the method in this area. (Asprone, *et. al.*, 2008).

From that perspective, this paper aims at reviewing the recent research works on the enhancements of the SPH method as it is applied in solid dynamics so as to identify the areas that still need further advances. Specifically, this study focused on identifying the principles of SPH Method and its application in solid dynamics, and to evaluate the effectiveness of the recent advances of the

SPH method for modelling in structures under dynamic loads, and suggesting the appropriate improvements on existing weaknesses of the method for achieving successfully modelling of structures under dynamic loads.
The paper mainly concentrated on a review on the principles of SPH method and its application in solid Dynamics, an evaluation of strengths and weaknesses of potential features in current SPH formulations, review of various ways for strengthening key features in SPH models for solid dynamics and giving conclusive remarks. Moreover, the execution of this work involved desk study on the originality of SPH method through reviews on relevant literature, pinpointing advances on SPH method as it is applied in structural dynamics by studying recent research works as well as a classical study on improved areas of SPH method in line with fundamentals of modelling and analysis of structures under dynamic actions.

GENERATIONS AND PRINCIPLES OF SPH METHOD

Generations of the SPH Method

Smoothed particle hydrodynamics (SPH) is a meshless modeling and analysis method which was founded by Lucy, (1977), and Monaghan & Gingold, (1977) as a modeling and analysis method in astrophysics domain for analyzing motions of stars in unbounded space. Then, Miyama, Hiyashi, & Narita (1984) applied this method in the determination of fragmentation in collapsing molecular clouds. Later, Benz *et. al.*, (2017) applied it in solving collision problems of star objects. This epoch is the first generation of the SPH method and can be termed as the classical SPH. In 1992, Monaghan successfully introduced it in analysis of fluid mechanics problems as the second generation of SPH method. Basing on those applications, interests then emerged on the applications of SPH method in other fields. Benz & Asphaug, (1995) and Belytschko *et. al.*, (1996) are among the first researchers who improved the classical SPH method and introduced it in solid mechanics as an alternative to the existing meshed methods. This era can be considered as the third generation of the SPH method. Monaghan (2005), introduced it in computer graphics and (Springel, 2005) applied it in cosmological simulations. This is the fourth generation of the SPH method.

Principles of SPH Method

According to Hu, (2021), Bagheri *et. al.*, (2023) and Pereira *et. al.*, (2017), any physical quantity of a continuum is discretized by particles, of which their positions are well defined by coordinates as shown in Figure 3. The process may start by estimating such a quantity on a cell corner which is made of four particles.

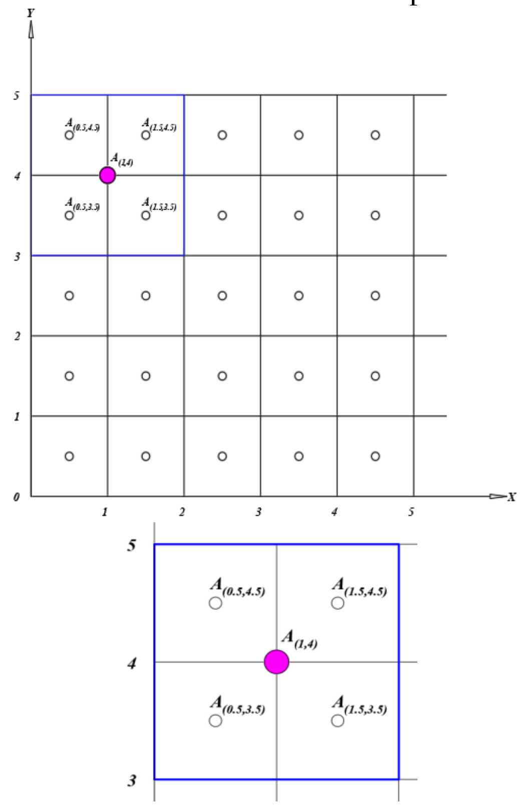

Figure 3: Discretization by particles in ordered pattern.

Let us assume that; $A_{0.5,3.5} = A_1$, $A_{1.5,3.5} = A_2$, $A_{0.5,4.5} = A_3$, $A_{1.5,4.5} = A_4$ and $A_{1,4} = A$. A particle quantity, m, at A with coordinates (1,4) as the sum of individual quantities of points A_1, A_2, A_3 and A_4 forming the cell with center at point A, can be defined as:

$$m_A = \sum_{k=1}^{4}(w_k\, A_k) \dots\dots\dots\dots\dots\dots\dots\dots\dots. (1)$$

In this case $\boldsymbol{w_k}$ is the weight contributed by individual particles to the quantity estimate and this depends on particle distance from the centre of the cell such that particle closer to the centre has higher weight contribution than the one which is far from the centre; and for consistency purposes, the summation of weight contribution from all particles must be equal to unit. From the above figure, therefore, $\sum_{k=1}^{4} w_k = 1$.

SPH method also performs in a continuum with particles allocated in disordered points as shown on Figure 4. In this case, approximation of partilcles' effects on any physical quantity is done first by selecting a set of particles of proposed uniform size to form boundary of the particles included in an estimate. This boundary in SPH language is known as the domain and in this paper its radius is denoted as 'h'and it is termed as the 'cut – off radius'. then, assigning weights to particles by applying special weight functions called kernels with respect to their distances from the centre of the domain. Again as the principle, consistency must be mantained on lumped effects of particle responses, such that, the summation of weights due to all particles in a domain must be equal to unit (1). It should also be noted that, particles existing outside the boundary contributes zero weights in a particular domain.

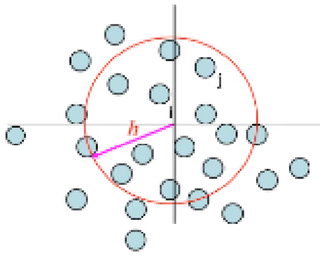

Figure 4: Discretization by particles in disordered pattern.

The lumped quantity, $A(\underline{r})$, of an element in a domain, Ω, can then be estimated by using an SPH interpolant as devepoed by (Monaghan, 1992) and (Hu, 2021) such that:

$$A(\underline{r}) = \int A(\underline{r}')\delta(\underline{r} - \underline{r})d\underline{r}' \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which, $A(\underline{r}')$ is the quantity of the particle,$'j'$ which is at the distance, $(\underline{r} - \underline{r}')$ from the centre of the boundary of the particles included in an estimate. This boundary in SPH language is known as the domain and in this paper the radius of that domain is denoted as 'h' as shown in Figure 4.

In equation (2) the term $\delta(\underline{r} - \underline{r}')$ is the dirac function that represents the weight function of individual particles involed in an estimate. Again, to maintain consistency in the domain, therefore;

$$\int \delta(\underline{r} - \underline{r})d\underline{r}' = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The term, $d\underline{r}'$ in equations (2) and (3) is the element of integration or simply the volume term. If we let $d\underline{r}'$ be equal to $\frac{m_j}{\rho_j}$ where m_j and ρ_j are mass and density of a particle 'j' at the distance $(\underline{r} - \underline{r}')$.. The SPH form of equation (3) can then be written as in equation (4) to define the same quantity, $A(\underline{r})$ but denoted as A_i and the weight function is given as W_{ij} This equation is called "The Basic Equation for SPH Formulation".

$$A_i = \sum_{N=1}^{N} W_{ij} \frac{m_j}{\rho_j} A_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Another significant issue in SPH formulations is the way of estimating derivatives which are important functions when solving a dynamic problem. Therefore, according to Monaghan, (1992) the first derivative of the an SPH formulation may be presented as:

$$\nabla_i . A = \sum_{N=1}^{N} W_{ij} \nabla \frac{m_j}{\rho_j} A_j \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

From equation (5) higher derivatives may be then be derived respectively.

APPLICATION OF SPH METHOD IN SOLID MECHANICS AND ITS ADVANCES

General Overview

The distinctive application of SPH method in analysis of structures under load actions can be well demonstrated by studying the deflection of a simple cantilever beam as a continuum that has been discretized in 2D finite particles as shown in Figure 5 below.

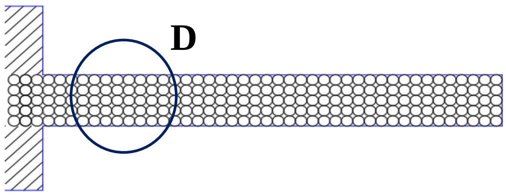

Figure 5: Particle discretization of a cantilever beam in SPH method.

Further, Figure 6 shows a magnified portion D of the beam in figure 5 such that a particle 'i' in the beam with initial position in the undeformed form is $\boldsymbol{C_0}$ of which its coordinates are (x_0,y_0) and its corresponding position vector $\underline{\boldsymbol{A}}$. In the deformed form after a time step, n, of a dynamic action, new position of the same particle will be $\boldsymbol{C_1}$ and its coordinates are (x_1,y_1) such that; $\boldsymbol{C_1}$ (x_1,y_1) = $\boldsymbol{C_0}$ + u, where u is travelled by a particle 'i' from C_0 to C_1 along both x and y axes.. The total displacement of the domain may be obtained as the vector summation of displacements of the individual particles forming the domain and basically is equal to vector sum of $\underline{\boldsymbol{A}}$ + $\underline{\boldsymbol{u}}$. (Naceur, et.al, 2014).

However, it should be well noted that, in SPH simulation, the value of any estimated quantity as the response of particles on the applied load in a particular domain is not constant for all particles but it decreases with the respect to particle distances from the center of the domain (Kayyer, et al., 2024). Now referring to the same Figure, the total deflection, in SPH form, $U(r)$ of the beam as a continuum is given by Naceur, et.al, (2014) as

$$U(r) = \sum_{i=1}^{N} u_j \, v_j W\left(r - r_j, h\right) \ldots\ldots\ldots\ldots.\ldots..(6)$$

where j is iterated over all particles, N = number of particles in a domain, v_j is the volume attributed to particle j, r_j the position, and u_j is the displacement of particle j at r_j. The volume v_j, is given as $\frac{m_j}{\rho_j}$, such that, m_j and ρ_j are mass and density of the particle j in the domain. W is the weight function as defined earlier (Naceur *et. al*., 2014).

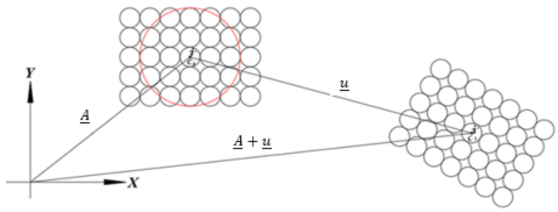

Figure 6: 2-D Particle displacement in SPH method.

With regard to solid continuum mechanics, as to-date, SPH method have been adapted with modifications or improvements in modelling and analysis of various structures that are subjected to either static or dynamic actions as discussed in subsequent paragraphs:

EVALUATION OF ADVANCES IN SPH METHOD FOR DYNAMICS OF STRUCTURES

Introduction of Kernel Functions as SPH Interpolant

Monaghan, (1992); introduces the replacement of the not-smooth dirac delta function with kernels as appropriate smooth weight function in an SPH interpolant to solve wave equations that are applicable in SPH problems. The researcher strongly recommended the application of the gaussian kernels of the third order and which are truncated at 2h, where 2h is the smoothing length that is defined as the width of the domain containing several particles. The original SPH formulation face criticalities of dealing with particles at edges as some of such particles can spill out of the domain boundary. Therefore, the estimation of their actual contribution to the physical quantity of the domain requires special treatment, the researcher however, did not propose the way of dealing with that issue.

Another critical problem in the method of which Monaghan (1992) did not address it, is the principle for setting the smoothing length in relationship to the number and size of particles forming the domain. This has significant effect to the response of the particles on external dynamic forces because particles with higher sizes will have big mass and thus result to higher inertia force as compared to smaller sized particles. Again, this issue needs special attention for the method to yield accurate response of continuum under dynamic action.

Nonlinear Analysis of Two-Dimensional Solid Structures

Naceur *et. al*., (2014) modified and efficiently adapted the SPH method for the analysis of two-dimensional structures which undergo nonlinearities in their geometry when subjected to dynamic actions. The modifications on the classical SPH were based on improving particle inconsistency and tensile instability for the structures under stress state as arises in Eulerian-based SPH problem which eventually cause high deformations of the continuum under loading. The first challenge was alleviated by introduction of modified kernels and their derivatives based on Taylor's series expansion while the second one was solved by proposing an SPH formulation that applies a total Lagrangian expression. The authors presented the SPH formulation for approximated quantity U_a as follows:

$$U_a = \sum_{b=1}^{N_b} u_b\, W_{ab} A_b \ldots\ldots\ldots\ldots\ldots\ldots.\ldots..(7)$$

where N_b refers to the number of neighboring particles b within the support domain and A is the area. The term W_{ab} is the weight function, which in the research refers to the B-spline function.

Applying the total Lagrangian formulation and to control the so-termed tensile instability, the authors adopted an approach which is given by

$$U_a = \frac{\sum_{b=1}^{N_b} u_b\, W_{ab} A_b}{\sum_{b=1}^{N_b} W_{ab} A_b} \ldots\ldots\ldots\ldots\ldots.\ldots..(8)$$

and the first derivative of the above function is therefore given as

$$(\nabla \cdot U)_i = \sum_{b=1}^{N_b} (u_b - u_a) \nabla W_{ab}\, A_b \ldots\ldots(9)$$

The methodology of the research involved modeling and analysis of a two-dimensional numerical model which was the roll up of a clamped beam. The geometry of the beam model is shown in Figure 7a where the length, *L = 100mm*, the width, *b = 5mm* and the thickens, *t = 5mm*. Material type for the beam is aluminum with density equal to $2700 kg/m^3$, elastic modulus, E = *73.4GPa* and a poison ratio, *v = 0.3*.

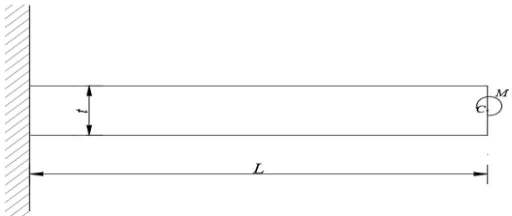

Figure 7a: Idealized beam with end moment.

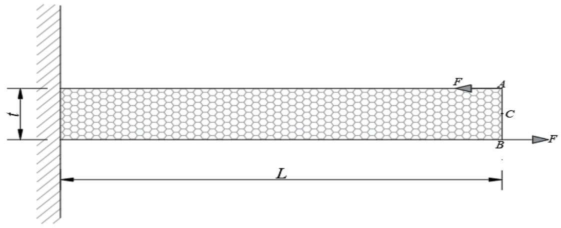

Figure 7b: Equivalent beam model.

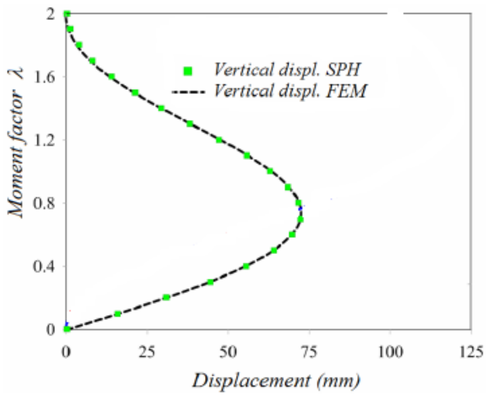

Figure 7c: Load-displacement relation between SPH and FEM.

The beam was discretized into 200 x 10 circular shaped particles and the idealized moment was generated at free end by couple of forces as shown in Figure 7b. Nonlinear analysis of the idealized beam was done after replacing the moment at C with a couple of forces AB as shown in Figure 7b.

For the purpose of following this work, it is important to recall back the principle of Euler beam such that:

The moment, *M*, of a beam is presented $M = \frac{EI}{\rho}$, and $\rho = \frac{s}{\theta}$, where *EI* is a beam stiffness, ρ is curvature, *s* is the length of arc (beam), θ, angle of curvature. Then, full curvature of a beam will occur when s = L and θ = 2π and in such a case therefore;

$$M = \frac{2\pi EI}{L})$$

Since the same Moment, *M* may be given as $M = Ft$, then, $Ft = \frac{2\pi EI}{L}$

This means that the force, *F*, which is required to generate the curvature at any angle, θ, can be obtained as:

$F = \frac{\lambda \pi EI}{Lt}$ where $\lambda = 0.0, 0.1, 0.2, \ldots, 2$

Again, it can be learnt from the methodology of this research, that analysis to determine vertical deflection at the free end while imposing various values of load factor, λ was done by using both improved SPH method with application of equations (8) and (9) and FEM where the solutions from both approaches as referred from Figure 7c show close results where the value of the expected maximum deflection when $\lambda = 2$ computed from SPH was +0.005mm and that of FEM was -0.006mm, meaning that in both cases the beam geometry was almost transformed to as circle. The conclusion to be drawn here is that, closeness of the results obtained from improved SPH method and FEM as the control method indicate the validity of the SPH method.

Analysis of Cable Structures

Dinçer & Demir, (2020) introduced the adaptation of classical SPH method in analysis of cable structures. They proposed a numerical model that represent a cable structure having only longitudinal stiffness in tension with the introduction of an artificial viscosity term and the damping parameter in SPH formulation for proper capturing of the cable behavior.

The momentum equation was then developed as follows:

$$\frac{dV}{dt} = \sum_{b=1}^{N} m_b \left(\left(\frac{\sigma_a^{ij}}{\rho_a^2} + \frac{\sigma_b^{ij}}{\rho_b^2} \right) + \pi_{ab}\delta^{ij} - \frac{\omega}{A} C_{ab} \right) \nabla W_{ab} + g_a \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

σ_a^{ij} and σ_b^{ij} are stress tensors while ρ_a and ρ_b are pressure on particles *a* and *b* respectively, *g* is the gravitational acceleration of a particle *a*, m_b mass of particle *b*, π_{ab} is the artificial viscosity and $\frac{\omega}{A} C_{ab}$ is the damping term.

This equation was then used to model and analyze a cable structure. However, the research work addressed neither the way of setting the smoothing length in relation to other parameters nor stated the advantages of the selected Wendland kernels over other kernels. Again, analysis of other potential solid structures, like beams under transverse vibration were not considered in this research.

Improvements on Loss of Particle Consistency in SPH Formulations

Sigalotti *et. al.*, (2021) carried out a comprehensive research on recent improvements on the consistency of the SPH method as multi-structural modeling and analysis technique. One of areas of interest in this research are the contents in chapters for particle inconsistency and corrective smoothed particle methods. In the first mentioned chapter, the authors addressed that particle inconsistency in an SPH doman is caused by error in the estimation of kernels and the procedure for particle descritization.

The authors stated that; if $\{f(x)\}$ is the smoothed estimate of a quantity $f(x)$ of a continuum and that W is the smoothing kernel function within the domain which is defined by the smoothing length, *h*; such that $W = 0$ except inside the domain whose radius is given by kh, where k is some integer that depends on the kernel. Now $W = 0$ *for* $||x\text{-}x'|| > kh$.When applying the Taylor's series expansion To ensure kernel consitency therefore, a higher order kernel has to be used suh that:

$$\int W(\| x - x' \|, h) d^n x = 1 \ldots\ldots\ldots\ldots.. (11)$$

The same principle can again be employed to derive the estimate of derivative and its moment as

$$\langle \nabla \mathrm{f(x)} \rangle = \int f(x') \nabla W(\| x - x' \|, h) d^n \ldots. (12)$$

The second error comes from the procedure for particle discritization which in fact is the function of the number of the particles, $n = \phi(n)$ around, the refered one and the way these particles are distributed (quasi or random ordered). For particles of a quasi-order, $\phi(n) \propto \frac{\log n}{n}$, while for randomly distributed particles, $\phi(n) \propto n^{-1/2}$**.** Again, is has been noted that the error in particle discretization can be reduced by making the smoothing length, *h*, smaller and the number of particles, *n* in a kernel domain, and number of all particles, *N* being larger**.** This means that, in the limit of $n \to \infty, h \to 0$, then $\frac{n}{N} \to 0$ becomes necessary for achieving full consistency. It is also cited that,

$h \propto N^{-1/\beta}$ and $n \propto N^{1-\frac{3}{\beta}}$. (13)
where $\beta = 3 + m/p$, and $½ \leq p \leq 1$. Now, for $m = 2$ and a random distribution (p = 1/2), β = 7 and $n \propto N^{0.57}$, while for quasi-ordered distributions (p = 1), β = 5 and $n \propto N^{0.4}$ and for an intermediate value, $\beta = 6$, $h \propto N^{-1/6}$ and $n \propto N^{1/2}$. If higher-order kernels ($m > 2$) are chosen, a stronger scaling of n with N would be required. The research thereafter studied various corrective methods on these challenges and recommended on the methods based on Taylor series expansions of the kernel approximations of a function and its derivatives in SPH for achieving any degree of consistency.

However, the research, warned that, such corrective schemes come at the price of involving the inversion of large matrices and hence, implying high computational cost for in terms of time required for simulations leading to a loss of numerical stability due to high possibility of large numerical errors in matrices.

In fact, authors reviewed the recent developments on application of SPH method as compared with the original one especially on weaknesses relating to kernels, particle discretization as sources of particle inconsistency being the critical shortfalls of the method and recommended the said improvements. Issues about the influence of other parameters like the relationship between material properties on setting of the smoothing length and the proper way of dealing with particles at or closer to the kernel boundaries and particular application on solids which is also one of the crucial issues were not broadly addressed in this research.

CONCLUSION AND RECOMMENDATIONS

From the findings of reviewed works, the paper agrees that, SPH method is more promising technique for modeling and analysis of various structures that are subjected to dynamic actions as compared to meshed methods and much advances have been done to improve weaknesses of the original SPH. However, such advances have only enabled application of this method in modeling and analysis of limited types of structures under dynamic actions such as bar element, cable systems and beam under static varying loads. Key areas that have been greatly improved include formulation of SPH problems by replacement of dirac delta function with kernel function as an appropriate SPH interpolant, expressions of derivative functions of SPH approximations and ways of improving particle consistency in SPH formulations.

Now; for the purpose of improving the applicability of the SPH method in solid dynamics, further research works is required on principles for selection of kernel function that fulfils all required interpolant requirements, criteria for setting of smoothing length in relation to other parameters such as particle size and density. More efforts have to be done on the procedures for setting the initial and boundary conditions of the kernel domain for SPH in solid dynamic problems.

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