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Energy Optimal Coverage Motion Trajectory Generation using a Fourth-order Motion Profile

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ABSTRACT

Industrial machines are widely used in manufacturing sector to manufacture several products to meet customer demands. Most of these industries runs all the time throughout a day leading to high operating cost. To cut costs and satisfy customer demand for precise products, industrial machines' motion generation is important in improving machine motion precision while using less energy. This study presents a coverage motion energy optimization which is generated by linear interpolation of each segment described by the fourth-order motion profile. The phase changes in the profile are attained with continuity of machine kinematic limits jerk, acceleration, and velocity, which are crucial to realize accurate motion. Genetic Algorithm is used to generate an optimal coverage motion using the convergence approach whereby the converged solution is selected as the final solution achieving minimum energy consumption. The simulation study is provided to illustrate the effectiveness of the proposed technique. The energy saving is about 9.27% when compared to unoptimized path. The approach can be utilized in several industrial machines employing coverage motion with the processes such as polishing, milling, laser cutting, and inspection

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INTRODUCTION

Energy saving and motion performance improvement in industrial machines' operation is vital to enhance productivity and lessen global issues such as energy shortages and environmental concerns (Cai, Liu, Zhou, & Xie, 2016) . There are several approaches proposed in literature for energy saving of industrial machines. Energy saving and motion accuracy improvement through controller designs in feed drive system is proposed (Msukwa, Nshama, & Uchiyama, 2020). However, their applications cannot be applied to industrial machine systems with an inaccessible controller. Moreover, to improve machine motion performance and energy saving, a number of motion trajectory planning are proposed. Trapezoidal velocity profiles are used to increase motion accuracy and energy saving as proposed in (Uchiyama, Honda, & Sano, 2014), and (Scalera, Carabin, Vidoni, & Wongratanaphisan, 2019). Trapezoidal velocity profiles mostly lead to machine excitation caused by abrupt changes in acceleration and infinite jerk values at the phase transitions (Martínez, Reséndiz, Prado, & Miguel, 2017), (Fang, Hu, Wang, & Peng, 2020). Methods such as input shaping and notch filters are employed to prevent machine excitation (Liu, Tsai, & Tang, 2020). However, the path geometry may be altered using filters and elongates the cycle times (Altintas, Verl, Brecher, Uriarte, & Pritschow, 2011), (Dumanli & Sencer, 2020). Furthermore, to avoid machine excitation by the use of trapezoidal profiles which generates infinite jerk, jerk limited acceleration profile (JLAP) is optimized to achieve accuracy motion and increase machine performance. Energy saving trajectory generation for point-to-point motion of industrial machines is proposed in (Uchiyama, Goto, and Sano 2015), and the JLAP is used to define the motion. In (Nshama, Msukwa, & Uchiyama, 2021), JLAP is optimized to achieve trade-off time and energy in point-to-point motion of industrial machine. Despite the use of JLAP, there are still inaccuracy in machine motion due to limited jerk leading to vibration, planning for jerk continuity is important as it improves machine performance (Tajima & Shamoto, 2018) (Wu, et al., 2022). Therefore, it is crucial to satisfy the machine kinematic limits and generate machine motion with jerk continuity while improving machine performance.

Furthermore, the path geometry is part of machine coverage motion which can be optimized to enhance machine performance and save energy. In (Feng, Chen, X., Zhang, J., Huang, Y., & Qu, Z., 2022), toolpath optimization is carried out for minimizing energy on industrial machine. However, the trajectory planning which enhance in achieving smooth motion is not considered. Therefore, for more improvement of industrial machine performance, it is important in implementing simultaneous trajectory generation and geometric path optimization.

METHODOLOGY

Trajectory Generation

To ensure smooth coverage motion with the jerk continuity, the fourth order motion profile is used for trajectory generation. Fourth-order motion profile is a trajectory profile that imposes a smooth transition of velocity, acceleration, and jerk for given predefined limits of velocity, acceleration, and jerk (Lambrechts & Steinbuch, 2005). The profile is created using fourth-order polynomial S-curve motion profiles that ensures smooth and continuity of the velocity, acceleration, and jerk as function of time. Figure. 1 shows the fourth-order motion profile for the jerk, acceleration, and velocity consisting of an acceleration phase from t_0 to t_7, a constant velocity phase from t_7 to t_8, and a deceleration phase from t_8 to t_{15} with the assumption that the time taken for the acceleration phase is equal to time for deceleration phase. The jerk profile consists of linear jerk, constant jerk, and zeros for some of time intervals forming fifteen segments of time periods. The trajectory of jerk profile is described as

$$\dddot{x}_k(t) = \begin{cases} j_{\text{lim},k},\frac{t}{T_l}, & t_0 \le t < t_1, t_{12} \le t < t_{13}, \\ j_{\text{lim},k}, & t_1 \le t < t_2, t_{13} \le t < t_{14}, \\ j_{\text{lim},k} - j_{\text{lim},k},\frac{t}{T_l}, & t_2 \le t < t_3, t_{14} \le t < t_{15}, \\ 0, & t_3 \le t < t_4, t_7 \le t < t_8, \\ & t_{11} \le t < t_{12}, \\ -j_{\text{lim},k},\frac{t}{T_l}, & t_4 \le t < t_5, t_8 \le t < t_9, \\ -j_{\text{lim},k}, & t_5 \le t < t_6, t_9 \le t < t_{10}, \\ -j_{\text{lim},k} + j_{\text{lim},k},\frac{t}{T_l}, & t_6 \le t < t_7, t_{10} \le t < t_{11}, \end{cases} \quad (1)$$

where T_l is the linear jerk period and $j_{lim,k}$ is the jerk limit for the k^{th} axis, and k is either x and y axis. To obtain position, velocity and acceleration, the equation (1) is integrated. The time intervals are calculated from the total time of linear segment which are divided into subinterval times while satisfying the kinematics limits of jerk, acceleration, and velocity as presented in equation (2) as;

$$\begin{aligned} &T_{l,1} = t_1 - t_0 = t_3 - t_2 = t_5 - t_4 = t_7 - t_6, \\ &T_{l,1} = t_9 - t_8 = t_{11} - t_{10} = t_{13} - t_{12} = t_{15} - t_{14}, \\ &t_{\text{cj},1} = t_2 - t_1 = t_6 - t_5 = t_{10} - t_9 = t_{14} - t_{13}, \dots\dots\dots\dots\dots\dots. (2) \\ &t_{\text{ca},1} = t_4 - t_3 = t_{12} - t_{11}, \\ &T_{\text{v},1} = t_8 - t_7. \end{aligned}$$

Where $T_{l,1}$, $t_{cj,1}$, $t_{ca,1}$ and $T_{v,1}$ are the linear jerk period, constant jerk period, constant acceleration/deceleration period, constant velocity period, respectively.

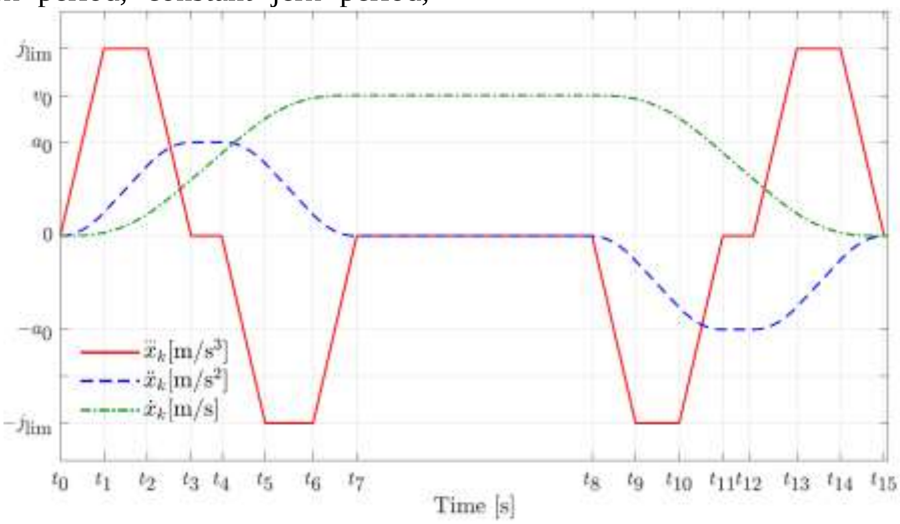

Figure 1: Fourth order motion profile

Industrial Feed Drive System Energy Model

With property that power is the function of velocity and acceleration, the industrial feed drive system energy model formulated by including velocity and acceleration and machine dynamics is used to optimize the energy consumption of the coverage motion. The energy consumption model used in this study is as used in (Halinga, Nshama, E. W., Schäfle, T. R., & Uchiyama, N., 2023) which incorporates the dynamics of the feed drive system as presented in equation (3). The energy is calculated for each linear segment, the total energy for the coverage motion is sum of energy of all segments. The expression of the energy model of the feed drive system is given as

$$P_k(t) = C_{1,k}\ddot{\boldsymbol{x}}_k^2 + C_{2,k}\dot{\boldsymbol{x}}_k^2 + C_{3,k}\dot{\boldsymbol{x}}_k \mathrm{sgn}(\dot{\boldsymbol{x}}_k) + C_{4,k} + C_{5,k}\ddot{\boldsymbol{x}}_k \mathrm{sgn}(\dot{\boldsymbol{x}}_k) + C_{6,k}\ddot{\boldsymbol{x}}_k\dot{\boldsymbol{x}}_k, \text{ for } k = \{x, y\}. \quad \ldots\ldots\ldots(3)$$

$$E = \int_{t_0}^{t_f} \left(|P_x(t)| + |P_y(t)|\right) dt, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

Where E is the total energy consumption for the industrial feed drive system to complete the motion time t_0 to t_f. $C_{i,k}$ is the i^{th} energy coefficient for the k^{th} axis. $P_k(t)$ is the power at time t for the x and y axes, respectively.

Motion optimization using Genetic Algorithm

The path and trajectory are optimized using genetic algorithm (GA) which is population-based algorithm using global search method in finding the optimal solution. The algorithm can be used to solve both constrained and unconstrained problem (Mirjalili, 2019). For given working surface as shown in Figure 3. there are several feasible solutions for the machine to complete the coverage motion in terms of geometric path and trajectory. It is important to find and select the best solution among several solutions which can be obtained by optimization method. The geometric path for the machine motion is formed by small linear segment when the machine moves from one discrete point to another. The machine trajectories: velocity, acceleration and jerk described on the linear segment are used to guide the machine motion while obeying to machine kinematics. The displacement information along linear segment is stored using *x* and *y* coordinates with grid points as reference. Having grid points (integers) and trajectories for the coverage motion, GA finds optimal and feasible path and trajectories between linear segment. In order to achieve the optimal solution, the problem is represented based on the algorithm execution, each grid is assigned an integer which is gene representation in algorithm and the trajectories are represented in real numbers.

Since the coverage motion problem is described above is combinatorial in nature, the optimization is executed by modifying the normal GA algorithm. The steps/flowchart for the GA algorithm is as shown in Figure 2, MATLAB® software is used for optimization. The execution starts with initializing the population using random and nearest neighbour method approach, followed by fitness evaluation and feasibility check. During fitness evaluation, equation (4) is used to calculate the value of each solution of the coverage motion while ensuring its feasibility, kinematic limits are satisfied and the time interval segments are above zero. Based on the fitness values, some of the solutions are selected to be parents for creating new offsprings, the roulette wheel method in combination with elitism is used to ensure parents/solutions with best fitness values are selected. From selected parents, the offspring are created using genetic operators which are order crossover and inversion mutation. Similarly to selection, during offspring creation, feasibility check is carried out to ensure that feasible solutions are obtained. Furthermore, after offspring creation, the algorithm selects new individuals to be new generation using roulette wheel and elitism and when the termination condition is met, the population becomes the final solution. The termination condition is the maximum number of iterations.

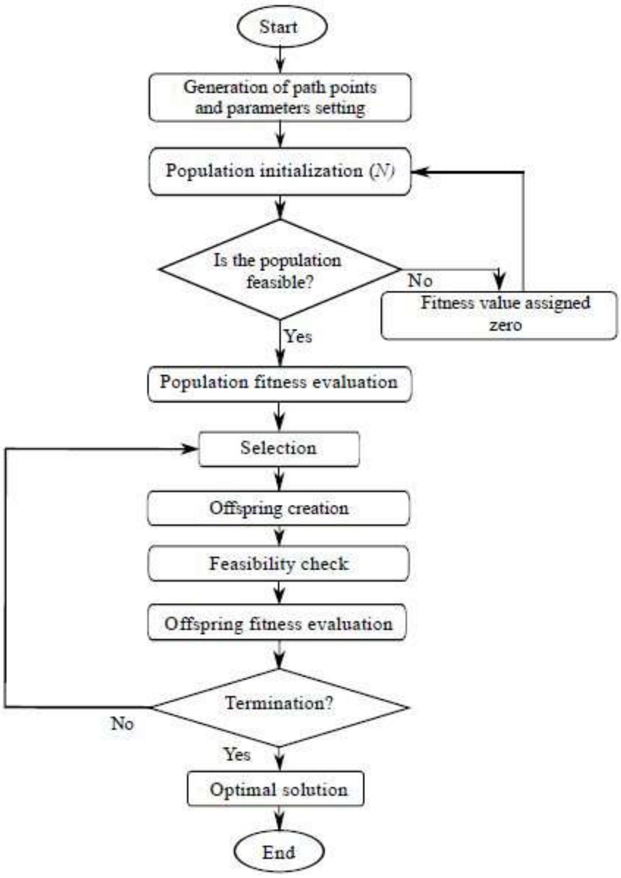

Figure 2: Illustration of optimization using GA with feasibility check.

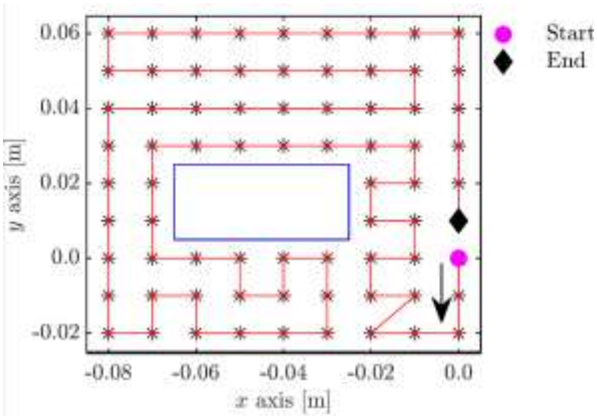

Figure 3: Working area for the coverage motion.

RESULTS

Optimization results

The 2D geometry shown in Figure 3 is used for optimization, the working surface is 90 mm x 90 mm in size with an island inside. The grid points are generated with distance 10 mm distance between neighbouring points. The machine kinematic limits velocity, acceleration, and jerk used are 0.08 m/s, 1 m/s^2, and 20 m/s^3, respectively. To optimize the GA, the parameters setting is: population size =100, the maximum number of iterations=200, crossover probability=0.8, and the mutation probability is set to 0.2. The algorithm is prepared and run to the maximum number of iteration (200) while converging to optimal solution. The converged optimal solution is achieved with the optimal energy of about 71.13 J equivalent to 9.27% energy saving compared to unoptimized solutions. Figure 4 shows the convergence plot during optimization. Furthermore, all the machine kinematic limits are obeyed and the jerk motion profile with trapezoidal is achieved as shown in Figure 5.

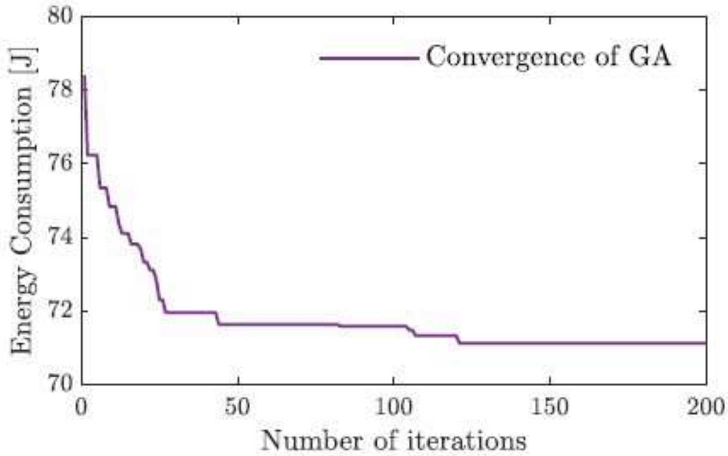

Figure 4: Convergence of the optimal solution using fourth-order motion profile.

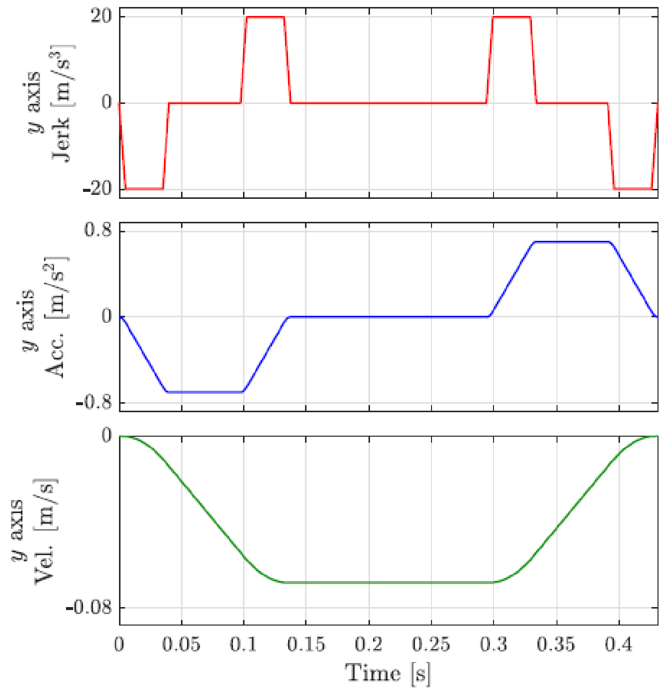

Figure 5: Generated motion profiles for the jerk, acceleration, and the velocity by fourth-order trajectory generation.

CONCLUSION

The study provides simulation method for obtaining energy optimal result for the coverage motion. Simultaneous trajectory and path optimization that provides the energy-optimal coverage motion is proposed achieving an energy saving of about 9.27% when compared to unoptimal result. The proposed approach can be used to machine operations such as polishing, milling, inspection and gluing. Further study is to implement trajectory optimization on curved surface unlike the plane surface considered in this study.

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