CONTROL SYSTEM DESIGN PROGRAM BASED ON CASCADE COMPENSATION

By

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Abstract

CAD is becoming more widely accepted in the design and analysis of control systems. A program has been developed for analyzing single-input single-output linear time-invariant control system. The application of the program for frequency domain design of cascaded phase lead and phase lag compensators is briefly discussed. Two examples to illustrate the program are presented.

Introduction

Frequency response methods are preferred in the design of control systems because they are easy to apply, and graphically they indicate clearly the type of change that is required to improve the system dynamic behavior. In most cases, the frequency response information for a system can conveniently be displayed on a Nyquist plot, Bode diagram, Nichol's chart or Smith chart.

When attempting to design a system using frequency domain, the designer must first decide what form of response is likely to be most acceptable. Should the system stability prove to be inadequate, appropriate compensator must be incorporated to eliminate the deficiency. The proper solutions of the system configuration and contents of the compensator, depend to a great extent on the experience and ingenuity on the part of the designer. Therefore, the frequency domain design is very much a trial and error proposition, and for higher order systems, calculations involved using the graphical method can be very tedious [1,4].

In this paper, the author introduces an inexpensive and efficient design procedure based on a computer program developed at the University of Dar es Salaam, Department of Electrical Engineering. The program is easy to use, runs interactively and supports an on line change of system parameters. The system being designed is presented as a transfer function in form of polynomial or factorized form to allow both the Nyquist and Bode plots to be implemented.
The paper explain how the developed computer program can be used for analysis and compensation of control system using Phase lead and phase lag cascade compensators.

When the Program was tested using previously developed transfer functions, the results obtained showed clearly how efficient and fast the analysis and design of control system can be accomplished compared to the use of manual graphical methods[1,2].

2. Basic design approach

In designing a control system, the main requirement is to achieve an adequate relative stability. Phase Margin and Gain margin are used to indicate how the locus of the open loop transfer function GH(jw) is close to the critical point -1+j0.

In order to ensure the stability of the system, it should be designed in such a way that both the phase margin and gain margin remain positive over a wide range of parameter variations. For satisfactory system response, the phase margin should be maintained between 30 and 60 degrees while the gain margin be greater than 6 dB.[2]

If the designed system fails to meet stability requirements, a compensator, which is an addition device should be inserted into the system to remedy the situation and ensure for the desired performance specifications. In most practical cases, compensation is essentially a compromise between steady-state error and relative stability. A Block diagram of a cascade compensated system is shown in figure 1.

2.2 Phase lead compensator

The primary function of the phase lead compensator is to reshape the frequency response curve to provide sufficient phase lead angle to offset the excessive phase lag associated with the components of the fixed system.
The general transfer function of a phase lead is given by:

$$G_c(s)=\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} \tag{1}$$

where $\alpha<1$.

2.3 Phase lag compensator

The primary function of a lag network is to provide attenuation in the high frequency range in order to give a system sufficient phase margin.

The general transfer function of a phase lag is given by:

$$G_c(s)=\beta\frac{s+\frac{1}{T}}{s+\frac{\beta}{T}} \tag{2}$$

where $\beta<1$.

3. Design program configuration

Communication with the computer is provided by pull-down menus that allow for the data input, system analysis and output. The language used is Turbo Pascal Version 4.0, allowing the program to be run on any IBM PC or 100% compatible IBM PC. A PC with a numeric Math coprocessor speeds the execution of the program.

3.1 Input

The open loop transfer function can be either in a factorized form or polynomial form. When the open loop transfer function is in the factorized form, the numerator should not exceed three factors. The same thing applies to the denominator. The maximum degree of every factor should be five. This means that the highest degree of either the numerator or denominator is fifteen. If the transfer function is in the form of polynomials, both the numerator and the denominator can have a maximum degree of ten. In either form, the user can design a higher order system. At this stage the coefficients of the open loop transfer function can be entered using the program screen editor.

3.2 System analysis

Various pull-down menus are provided which allows the designer to investigate the response behavior of uncompensated and compensated system. Different menus are shown in figure 2. Other peripheral devices for output of results, bulk storage, as well as retrieval of data from memory are linked to the program. The stored coefficients of the transfer function are retrieved back on the program screen editor when you run the program latter.

3.3 Output

The output menus supporting the program provide for the on-line screen display of system parameters being investigated (e.g. frequency, magnitude and phase angle, gain margin, phase margin, phase and gain crossover frequencies etc). Bode and Polar plots of uncompensated and compensated system can be either displayed on the screen or sent to Hewlett Packard 7475A or 7440A plotter for the hard copy printout. The user can quit the program at any stage by using the quit option from any menu.

4. Program test run

In this part of the paper, use is made of the developed program to show how it can be applied in control system design and compensation. To be able to demonstrate this, some transfer functions of two control system, both requiring compensation are examined.

The first control system is investigated using phase lag and the second system is investigated using a phase lead cascade compensation networks. The hard copy of the corresponding Polar and Bode plots of the uncompensated and compensated system are also presented.

4.1 Phase lag compensation

A control system with an open loop transfer function given by

$$G_p(s)H(s) = \frac{K}{s(1+0.1s)(1+0.2s)} \tag{3}$$

is to be compensated by means of cascade phase lag compensator to have a phase margin of at least 45 degrees and a static velocity error coefficient K_v of at least 20 sec^{-1}.
Find the gain margin, phase margin, gain crossover frequency and phase crossover frequency before and after compensation. Draw the Bode plot and Polar plot for each case.

Solution:

By definition, the velocity error coefficient is given by:

$$K_v = \lim_{s \to 0} sG(s)H(s) \tag{4}$$

Using the above formula we get that K=20. Using the developed program and entering the coefficients of the open loop transfer function we get the following results:

(a) **Uncompensated system:**

Using menu 3 option 3 of the program we get:

Gain margin	= -2.478 dB
Phase margin	= -7.508 degrees
Gain crossover frequency	= 8.222 rad/sec
Phase crossover frequency	= 7.161 rad/sec

This indicates clearly that the system is unstable and the corresponding Bode plot and Polar plot for uncompensated system are shown in fig 3 and 4 respectively.

(b) **Compensated system**

Using menu 4 option 2 allows the user to select a phase lag compensator network and after specifying the required phase margin and the safety factor, the program produces the following results:

β	= 0.1349
1/T	= 0.2399
β/T	= 0.0324
T	= 4.1687

Therefore according to equation 5 the transfer function of phase lag compensator is given by:

$$G_c(s) = 0.1349\left(\frac{s+0.2399}{s+0.0324}\right) \tag{5}$$

The open loop transfer function of compensated system becomes:

$$G_p(s)H(s)G_c(s) = \frac{134.87(s+0.2399)}{s(s+5+(s+10)(s+0.0324)} \tag{6}$$

Menu 6 option 2 allows the user to analyse the resulting compensated system and parameters obtained are:

Gain margin	= 14.520 dB
Phase margin	= 45.943 degrees
Gain crossover frequency	= 2.427 rad/sec
Phase crossover frequency	= 6.998 rad/sec

From the above results with the Bode and Polar plots for compensated system shown in figure 5 and 6 respectively, the system is stable and meets both the steady-state and the relative stability requirements.

4.2 Phase lead compensation

A control system with an open loop transfer function given by

$$G_p(s)H(s) = \frac{K}{0.25s^2+s} \tag{7}$$

is to be compensated by means of cascade phase lead compensator to have a phase margin of at least 45 degrees and a static velocity error coefficient of at least 100 sec^{-1}.
Find the gain margin, phase margin, gain crossover frequency and phase crossover frequency before and after compensation. Draw the Bode plot and Polar plot for each case.

To investigate the system, application is made using the above approach. Using equation 4 we get K=100.

Using the program we get the following results:

(a) Uncompensated system:

Using menu 3 option 3 we get the following:

Gain margin = Infinity
Phase margin = 11.34 degrees
Gain crossover frequency = 20.184 rad/sec

Bode and Polar plots are shown in Fig. 7 and 8 respectively. Since the phase margin is not satisfactory, a compensator is needed.

(b) Compensated system

Using Menu 5 option 1 we get the following parameters of Compensator:

α = 0.2310
1/T = 13.7363
$1/\alpha T$ = 60.0114
T = 0.0728

According to equation 3 the transfer function of Phase Lead is given by:

$$G_c(s) = \frac{s+13.7363}{s+60.0114} \tag{8}$$

To compensate for the attenuation due to the lead network, we increase the amplifier gain by a factor of $1/\alpha$. The open loop transfer function of compensated system becomes:

$$G_p(s)H(s)G_c(s) = \frac{1731.9(s+13.8601)}{s(s+4)(s+60.0114)} \tag{9}$$

Parameters for compensated system are:

Gain Margin = Infinity
Phase Margin = 47 degrees
Gain Crossover frequency = 29.174 rad/sec

Bode and Polar plots for compensated system are shown in figure 9 and 10 respectively. From the above results, the system meets both the steady-state and the relative stability requirements.

Conclusion

The paper has discussed a computer program developed for analysis and compensation of control system. It allows the user to interactively carry out design exercises with much ease and lot of flexibility to introduce changes very quickly. For those who have already used the program (students and staff in Electrical Engineering Department) are of the opinion that the program is superior to manual graphical methods because of its user-friendly, accuracy and time saving.

Two examples have been provided to illustrate the program. The program is being expanded to offer also the following services:

- Root Locus plots
- dB-magnitude versus Phase angle plot.
- Analysis of closed loop frequency response for unity and non-unity feedback system without using Nichol's chart graphical method.
- Time domain response

References

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4. Schwarznbach, J. and Gill,K.F., System Modelling and Control, 2nd edition, Edward Arnold (1984).
5. Hostetter,G.H., Savant,C.J., Stefani, R.T., Design of Feedback Control Systems, Holt-Saunders International Editions (1982).
6. D'azzo,J.J, Houpis, C.H; Linear Control System Analysis and Design, McGraw-Hill (1975).

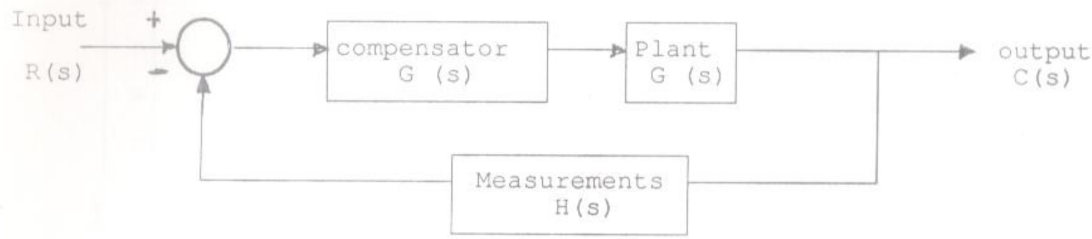

Fig.1. Cascade compensation

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FORM OF TRANSFER FUNCTION

  1. FACTORIZED FORM
  2. POLYNOMIAL FORM
  3. TERMINATE PROGRAM
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Fig.2a. Menu 1

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AVAILABLE SERVICES:

1. ANALYSIS OF UNCOMPENSATED SYSTEM
2. ANALYSIS OF COMPENSATED SYSTEM
3. LEAVE THE MENU
4. TERMINATE PROGRAM
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Fig.2b. Menu 2

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   ANALYSIS OF UNCOMPENSATED SYSTEM

AVAILABLE SERVICES:

 1. ENTERING POLES AND ZEROS
 2. DISPLAY ANGLE AND MAGNITUDE VALUES
 3. DISPLAY PERFORMANCE PARAMETERS
 4. DRAW BODE PLOT ON THE SCREEN
 5. DRAW POLAR PLOT ON THE SCREEN
 6. DRAW BODE PLOT ON THE PLOTTER
 7. DRAW POLAR PLOT ON THE PLOTTER
 8. LEAVE THE MENU
 9. TERMINATE PROGRAM
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Fig.2c. Menu 3

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   TYPE OF COMPENSATION

AVAILABLE SERVICES:

   1. PHASE-LEAD COMPENSATOR
   2. PHASE-LAG COMPENSATOR
   3. LEAVE THE MENU
   4. TERMINATE PROGRAM
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Fig. 2d. Menu 4

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PHASE-LEAD COMPENSATION

AVAILABLE SERVICES:

1. DISPLAY PARAMETERS OF CONTROLLER
2. DISPLAY ANGLE AND MAGNITUDE VALUES
3. DISPLAY PERFORMANCE PARAMETERS
4. DRAW BODE PLOT ON THE SCREEN
5. DRAW POLAR PLOT ON THE SCREEN
6. DRAW BODE PLOT ON THE PLOTTER
7. DRAW POLAR PLOT ON THE PLOTTER
8. lEAVE THE MENU
9. TERMINATE PROGRAM
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Fig.2e. Menu 5

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PHASE-LAG COMPENSATION

AVAILABLE SERVICES:

1. DISPLAY PARAMETERS OF CONTROLLER
2. DISPLAY ANGLE AND MAGNITUDE VALUES
3. DISPLAY PERFORMANCE PARAMETERS
4. DRAW BODE PLOT ON THE SCREEN
5. DRAW POLAR PLOT ON THE SCREEN
6. DRAW BODE PLOT ON THE PLOTTER
7. DRAW POLAR PLOT ON THE PLOTTER
8. LEAVE THE MENU
9. TERMINATE PROGRAM
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Fig.2f. Menu 6

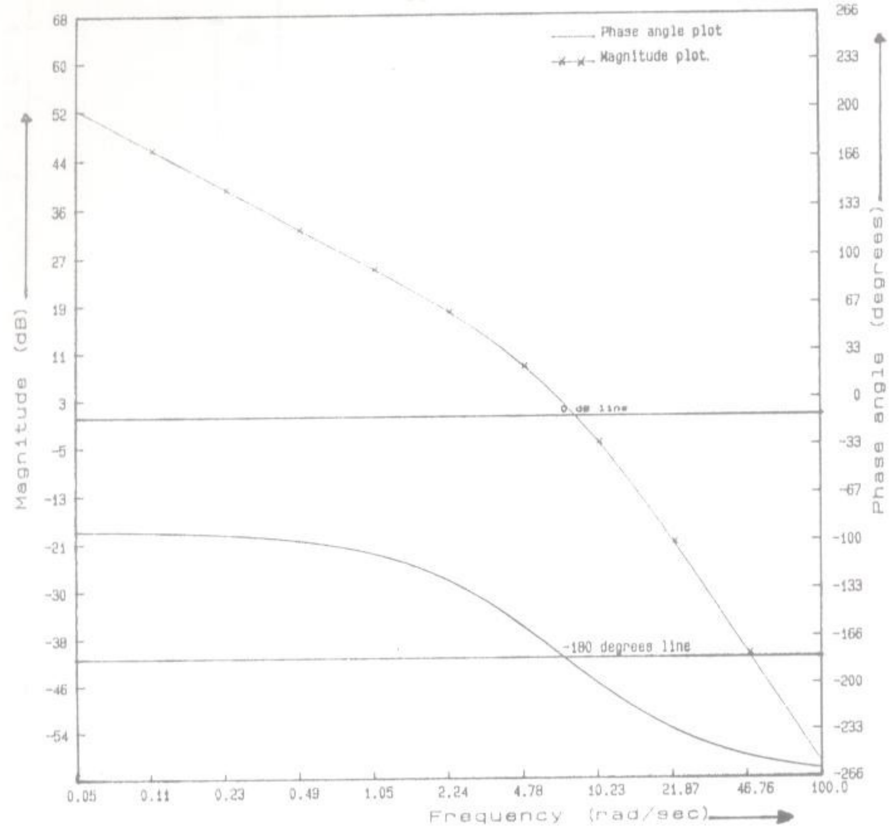

Fig.3. Bode Plot for the uncompensated system (Phase lag compensation).

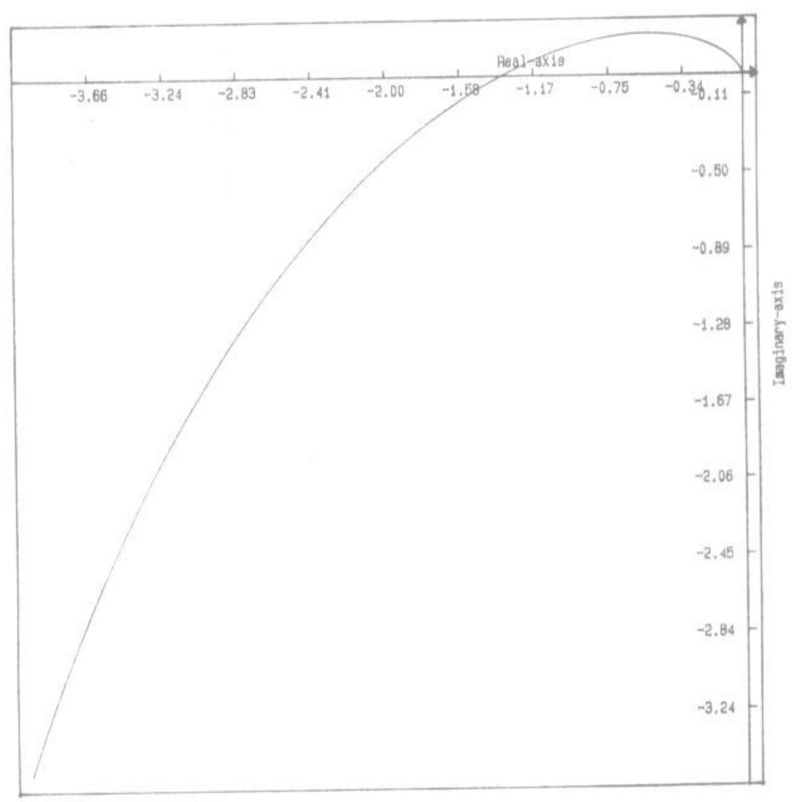

Fig.4. Polar Plot for the uncompensated system (Phase lag compensation).

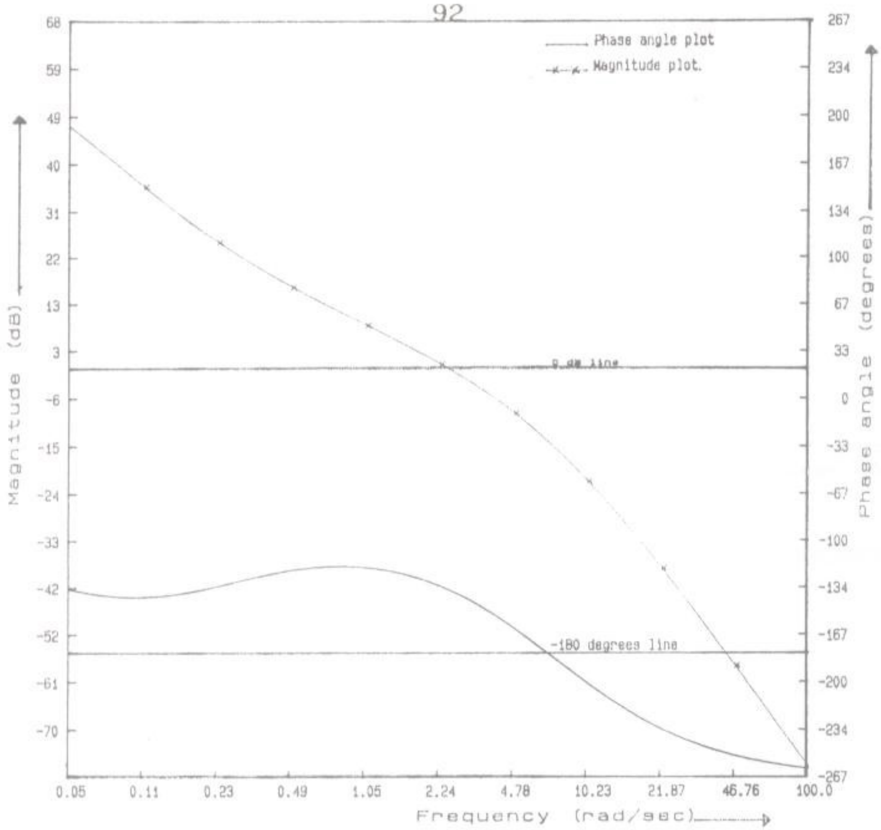

Fig.5. Bode Plot for the compensated system (Phase lag compensation).

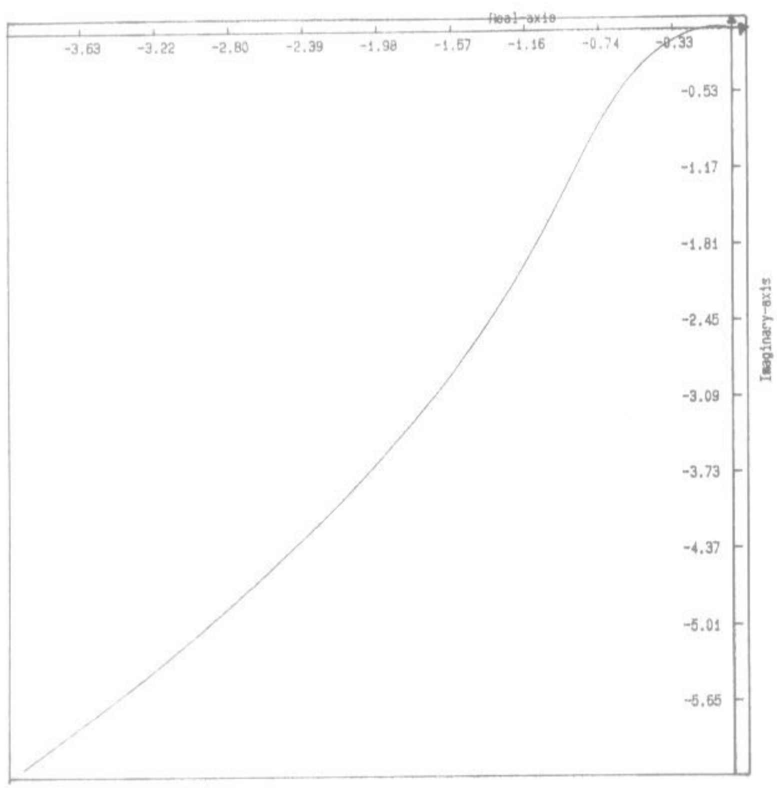

Fig.6. Polar Plot for the compensated system (Phase lag compensation).

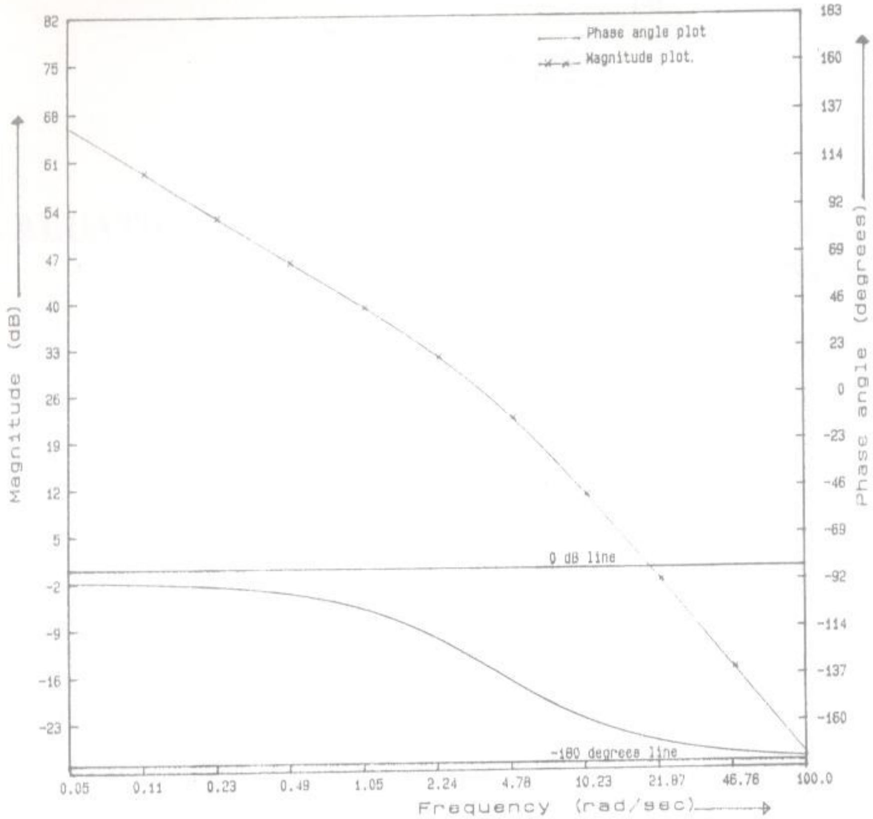

Fig.7. Bode Plot for the uncompensated system (Phase lead compensation).

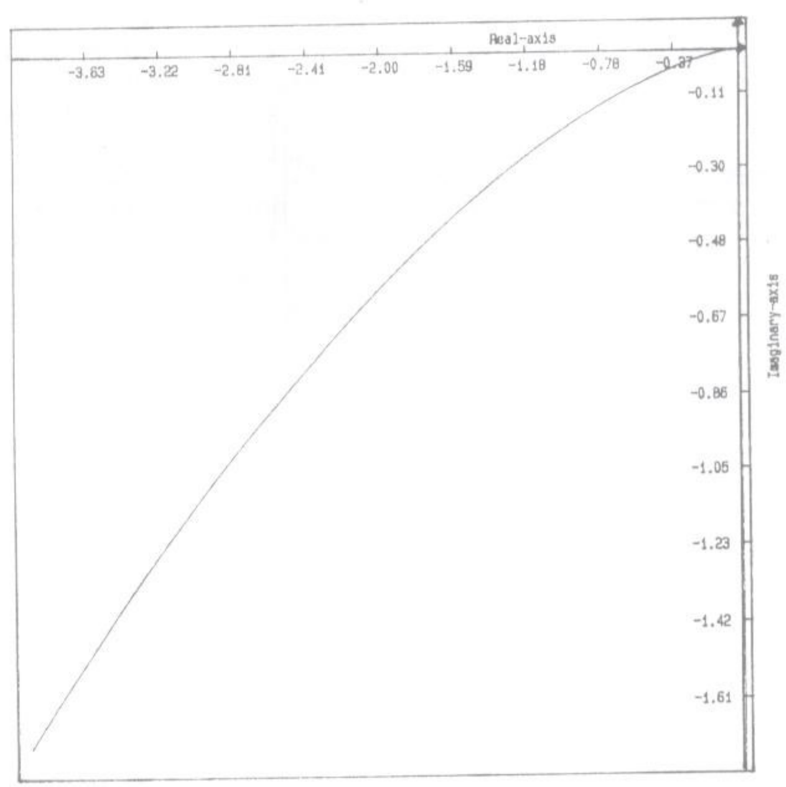

Fig.8. Polar Plot for the uncompensated system (Phase lead compensation).

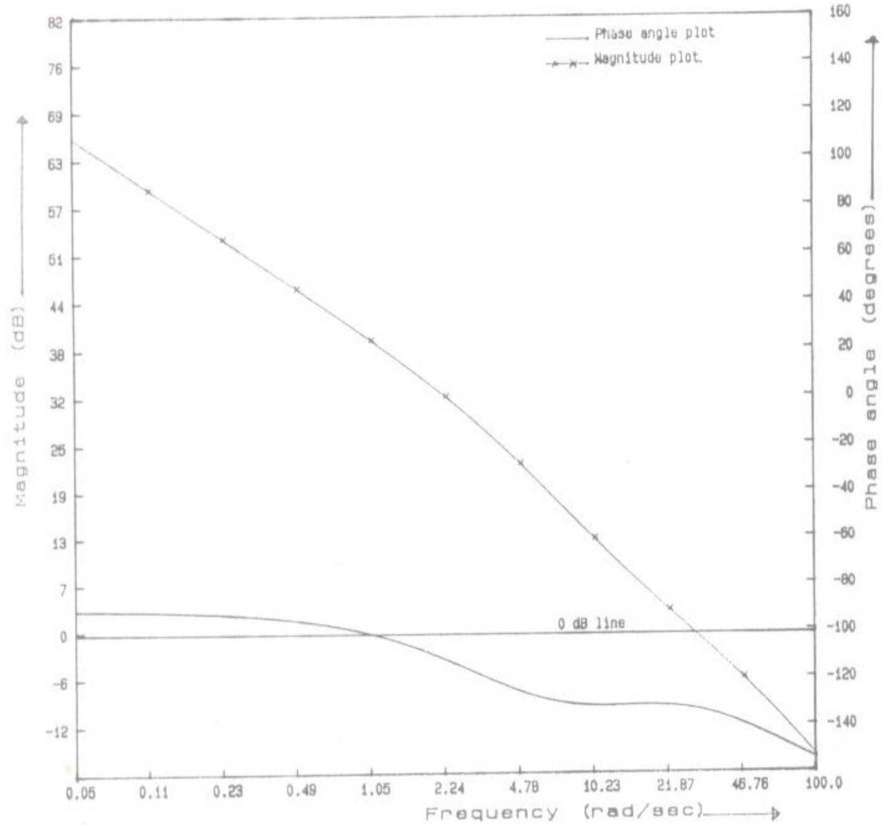

Fig.9. Bode Plot for the compensated system (Phase lead compensation).

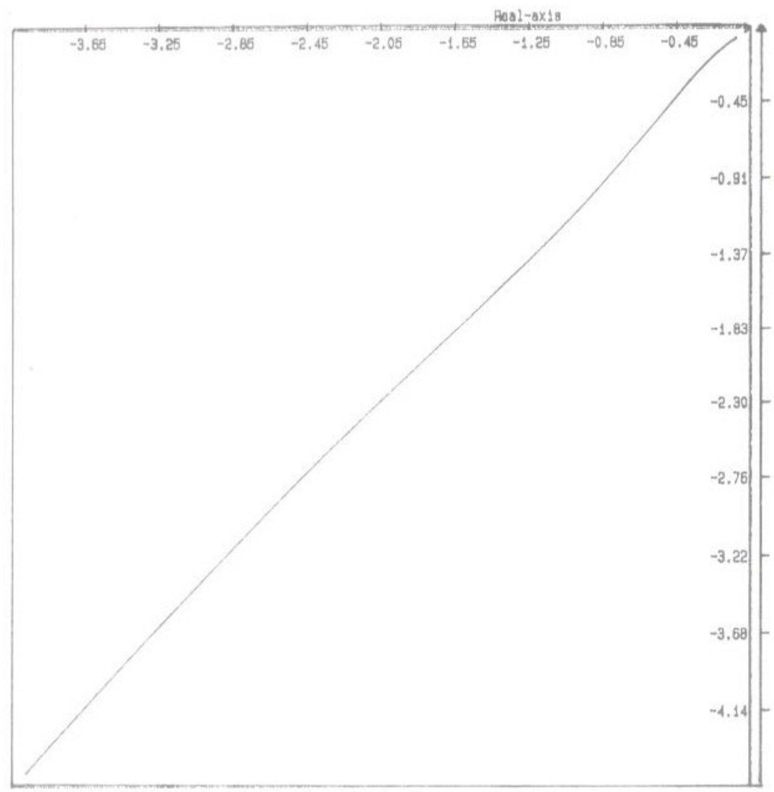

Fig.10. Polar Plot for the compensated system (Phase lead compensation).