EFFECT OF REDUCED PLATE WIDTH ON THE STRUCTURAL BEHAVIOUR OF A TRIANGULARLY FOLDED PLATE BARREL VAULT

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ABSTRACT

Various methods of analysis have been employed to predict the structural behaviour of the Triangularly Folded Plate Barrel Vault, with conflicting results. A review of past analytical and experimental investigations carried out on the structure show that theoretical analysis based on a smaller effective width rather than the overall plate width is capable of closely predicting the deflection behaviour of the structure.

This paper presents an experimental study of the behaviour of a triangularly folded plate barrel vault with physically reduced plate width.

It is found that physical reduction of the plate width to correspond to 'theoretical' effective width weakens the structure, by making it less stiff and highly susceptible to buckling at high loads.

The results also confirm earlier postulates that the central part of a plate serves to stabilize the folds and helps to prevent torsional buckling at higher loads. The results are useful when considering the design and construction of cost-effective triangularly folded plate structures.

1.0 INTRODUCTION

The triangularly folded plate barrel vault (Fig. 1), which is a type of cylindrical shell structure, has the advantage of simplicity in fabrication and erection and the additional advantage of potentially being a portable and demountable structure. The geometry of individual plates has a direct influence on the overall geometry of the structure and hence an influence on the overall stiffness.

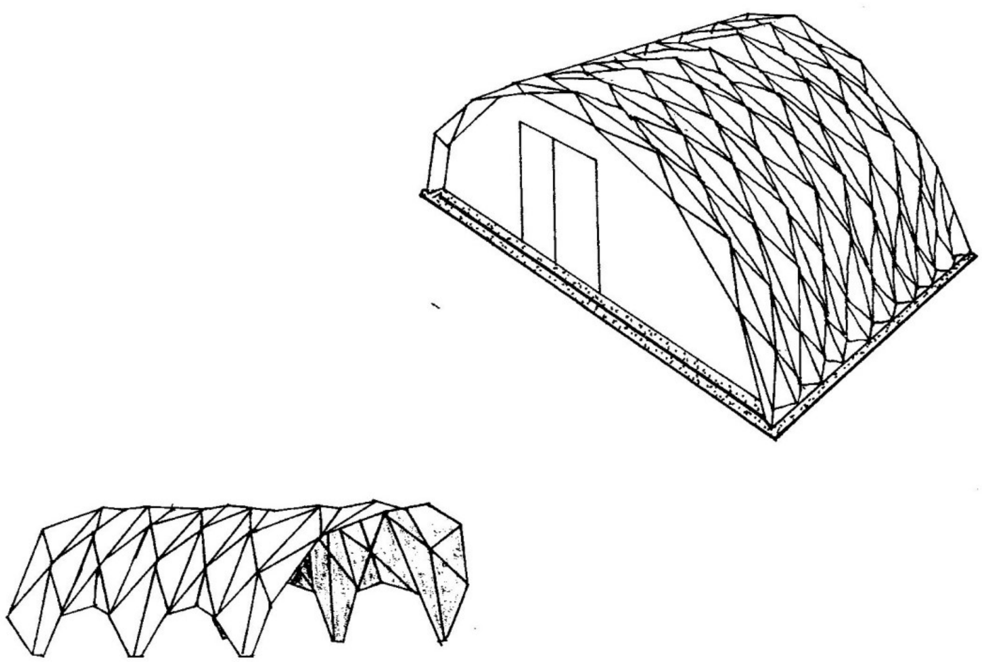

Fig. 1 Triangularly folded plate barrel vault

Various methods of analysis of the triangularly folded plate barrel vault ranging from simple approximate methods to finite element computer methods have been used to predict the behaviour of the structure, with conflicting results[1-4]. However, a review of experimental investigations already carried out on the structure show that theoretical analysis based on a smaller effective width rather than the overall plate width, using the combined Plate- and Arch-action method[2], is capable of closely predicting the deflection behaviour of the structure. Benjamin[2] and Zhidkov[5] have further suggested that the central portions of the triangular plates are structurally less useful than the folds, that they act merely as covering membranes which serve to transfer load to the folds only when the structure is highly stressed.

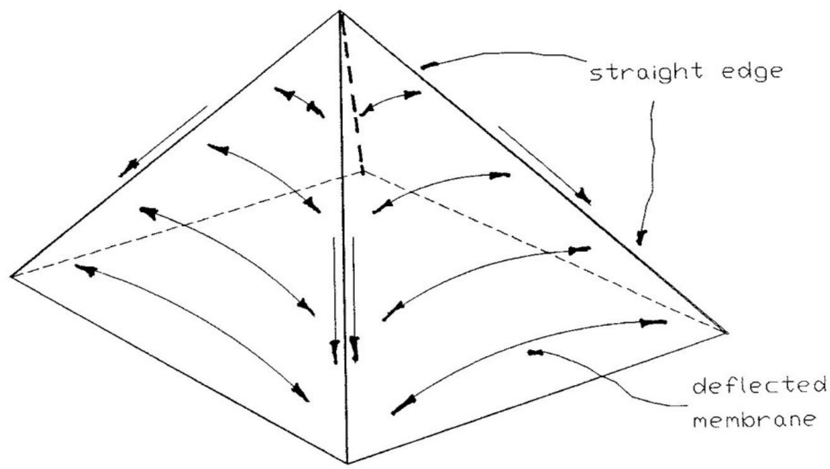

Fig. 2 Membrane action in folded plate Pyramid

The theory of plate behaviour in folded plate structures also presumes that plate buckling at low loads causes the latter to be carried by the folds alone, therefore literally reducing the structure to a skeleton[5]. This has led to further suggestions that analysis of a single-skin triangularly folded plate barrel vault may be reduced primarily to the analysis of a skeletal spatial frame[5]. Gilkie and Robak[6] have shown experimentally that physical reduction of the plate width is possible without significantly affecting the deformation behaviour of a hexagon-based folded plate pyramid, however only up to a certain stress level.

The work presented in this paper was undertaken to study the behaviour of a triangularly folded plate barrel vault with physically reduced plate width, using methods of physical model analysis.

2.0 EXPERIMENTAL INVESTIGATIONS

The objective of the study was to find out how physical reduction of the plate width affects the structural behaviour of a triangularly folded plate barrel vault. This objective was attained by designing and testing four models of the structure, each with a different plate width as shown in Fig. 3.

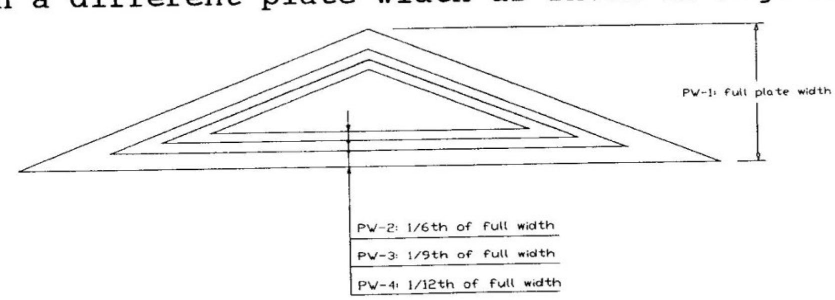

Fig. 3 Variations of plate widths

The reduced widths were achieved by cutting off the appropriate central portions of individual plates.

2.1 PHYSICAL MODELS

The experimental models were designed to have the geometrical configuration shown in Fig. 5. The basic model is a triangularly folded plate barrel vault with a neutral axis span of 1331.3mm, a maximum span of 1348.2mm and a maximum height of 674.1mm. It is composed of identical rhomboidal units made of two 3 mm thick isosceles triangular plates of base length 300 mm, and a base angle of 22.5°. The fold angle between adjacent plates was 148.43°. The basic unit is illustrated in Fig. 4.

The designations of the tested models are presented in Table 1.

Table 1: Model designations

MODEL	PLATE WIDTH
PW1	Full Plate Width
PW2	1/6th of Full Plate Width
PW3	1/9th of Full Plate Width
PW4	1/12th of Full Plate Width

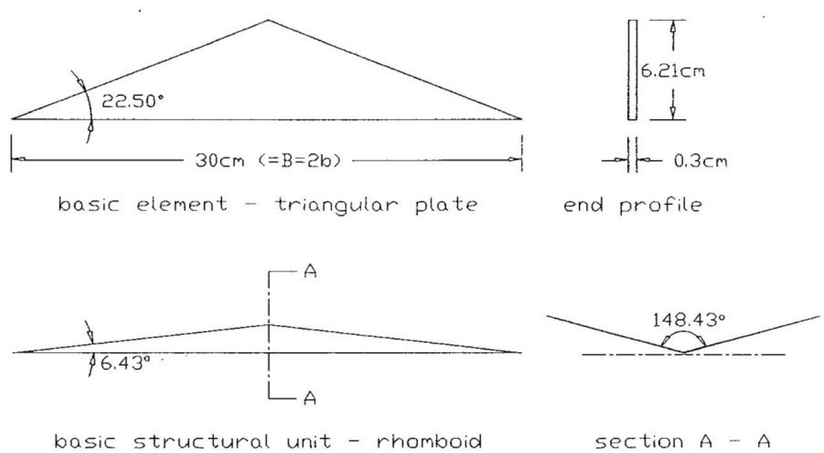

Fig. 4 Basic unit of experimental model

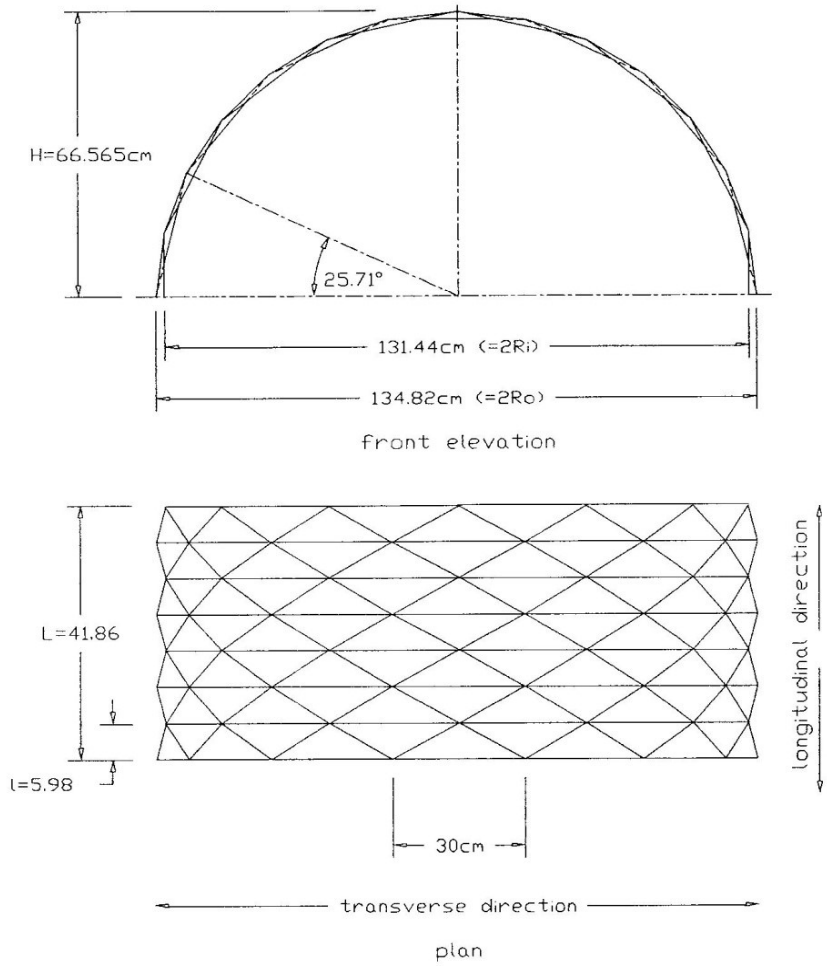

Fig. 5 Elevation and plan of experimental model

2.2 MODEL MATERIALS AND FABRICATION

The basic elements of the model were cut from a 3 mm thick sheeting of 'FOAMEX[TM]', a foamed rigid polyvinil chloride (PVC). Laboratory tensile tests found the material to have an elastic modulus of 1081 N/mm^2 and a Poisson's ratio of 0.3826. It was estimated that the material had a yield limit in tension of 5.74 N/mm^2. The joints between elements were provided by a 'BOSTIK[TM]' epoxy resin, which had a modulus of elasticity of 2576 N/mm^2 and a Poisson's ratio of 0.4382.

Elements of the structure were assembled on a flat surface so as to produce a developed plan view of the model. They were jointed together on one side by a muslin-backed clear 'BOSTIK[TM]' glue tape to form flexible joints. The whole assembly of plates was then mounted on a made-to-measure semi-cylindrical timber formwork, which gave the model its geometrical form and overall dimensions. The joints were filled with 'BOSTIK-2001[TM]' epoxy glue which yielded rigid joints after hardening.

2.3 EXPERIMENTAL PROCEDURE

(a) Test set-up

All models were tested on a specially constructed steel testing frame[7]. Foundation for the model was provided in the form of two channel troughs, separated by a centre-to-centre distance equivalent to the neutral axis span of the models. Continuous fixed support on each longitudinal base of the model was achieved by fixing the base in the troughs with ARALDITE[TM] casting resin (Fig. 6).

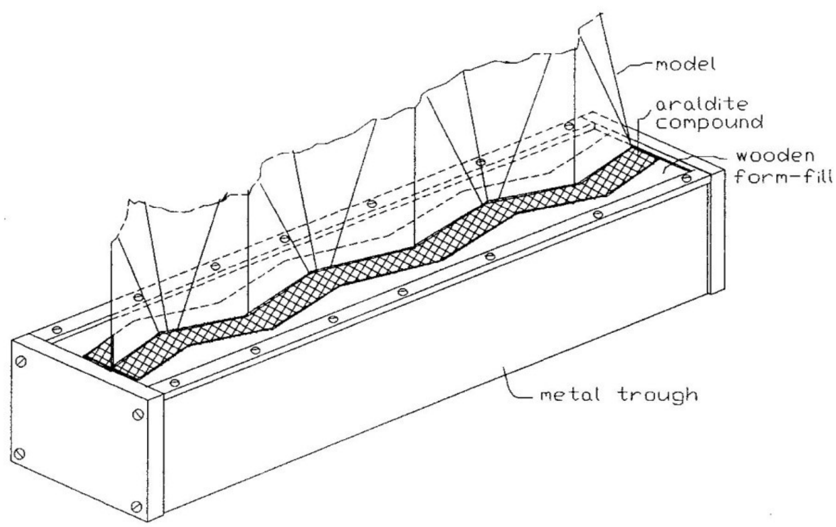

Fig. 6 Model 'foundation'

(b) Instrumentation

Electrical resistance strain gauge rosettes were used to measure strains at selected points on the top and bottom surfaces of the model[7]. A quarter-bridge resistive network connection of the gauges was employed, with all gauges sharing a common dummy to counter thermal effects of the surrounding ambient. An automatic semi-continuous datalogging system was used to record test results in microstrains.

Vertical and horizontal deflections at selected positions[7] were measured by means of mechanical dial gauges with divisions of 0.0254 mm.

(c) Loading

The models were tested for three loading cases[7], designated 1(V), 2(V) and 3(H), comprising of vertical and horizontal line loads as shown in Fig. 7.

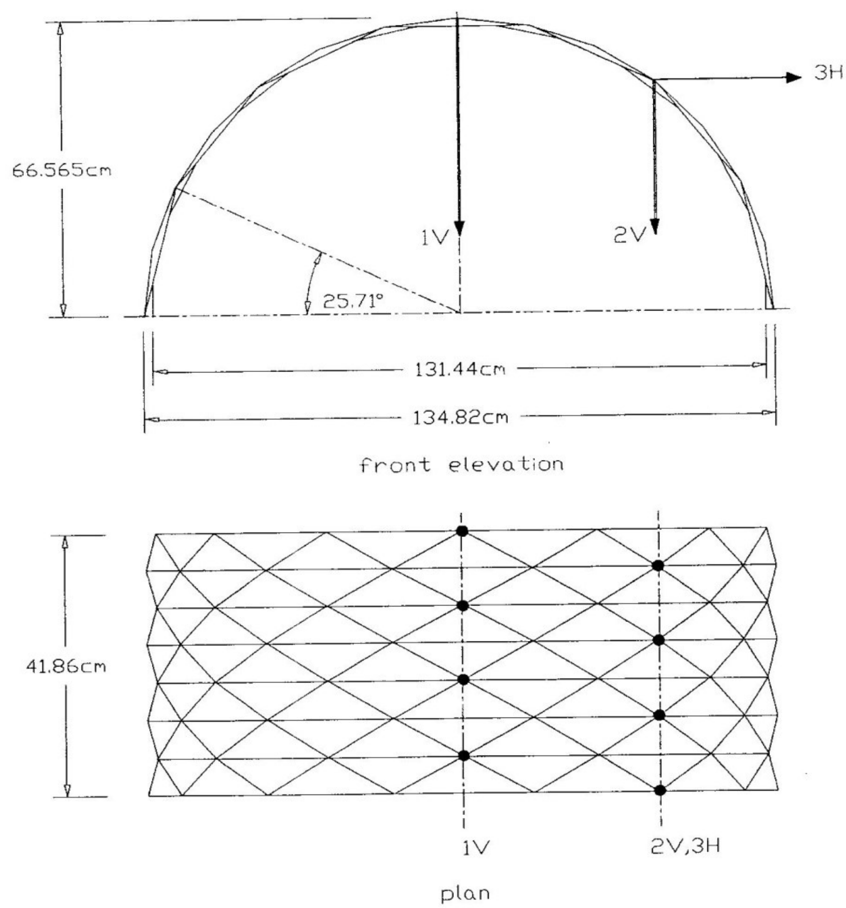

Fig. 7 Loading cases

(d) Test Procedure

Strain and deflection measurements on the model were taken before and after the load was applied, in order to eliminate errors due to external effects and initial conditions.

3.0 RESULTS

3.1 DEFLECTIONS

The deflection diagrams in Fig. 8 show the influence of physically reducing the plate width[7]. It is shown that while the load remains constant deflections increase as the plate width decreases. The model with 1/6th of the full plate width shows the least increase of deflections compared to the other variations. Fig. 9 shows relationships between relative deflections and relative load, with the initial increment of load and corresponding deflections of the full-width plate model taken as reference values[7].

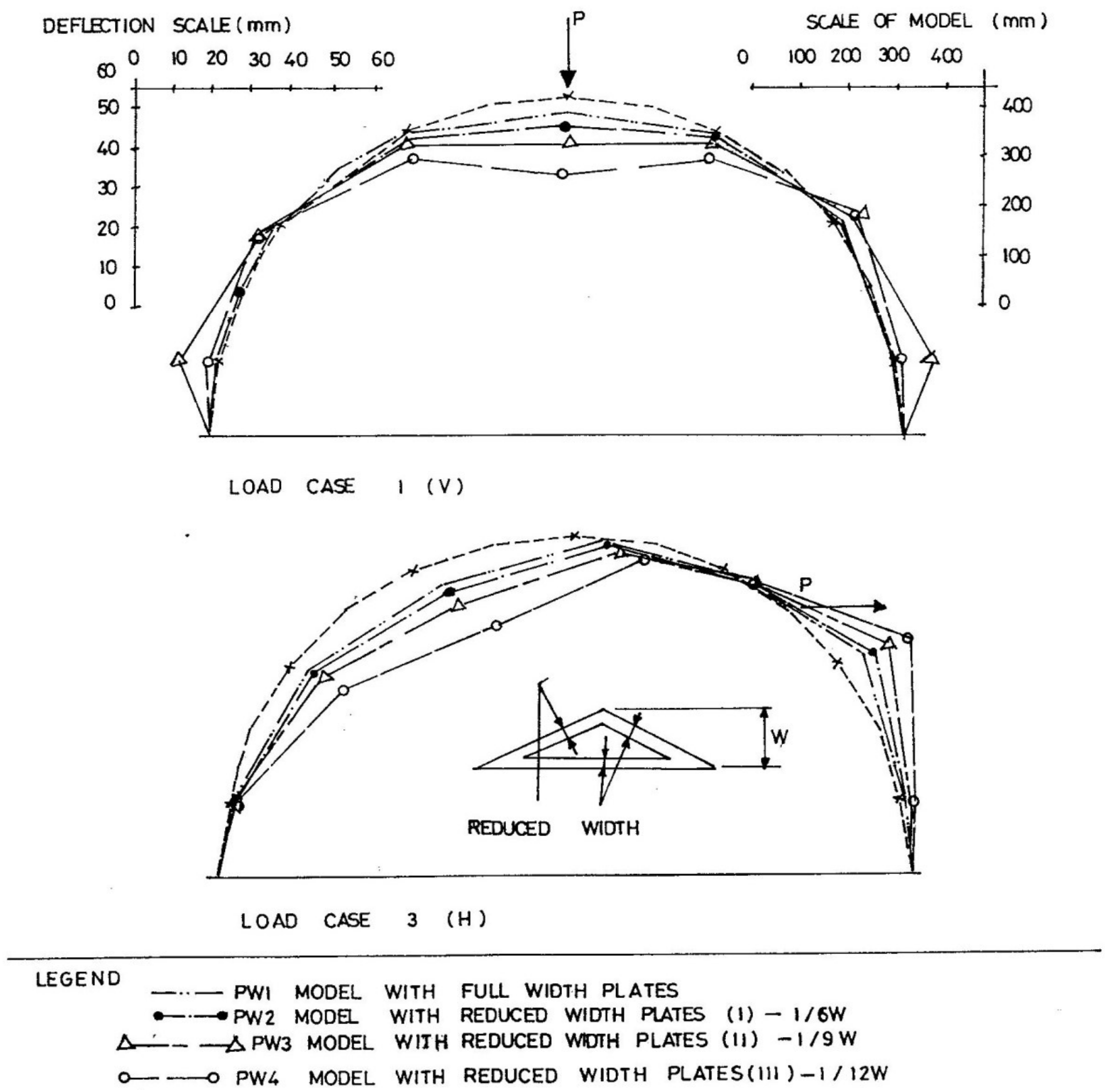

Fig. 8 Influence of plate width reduction on model deflections

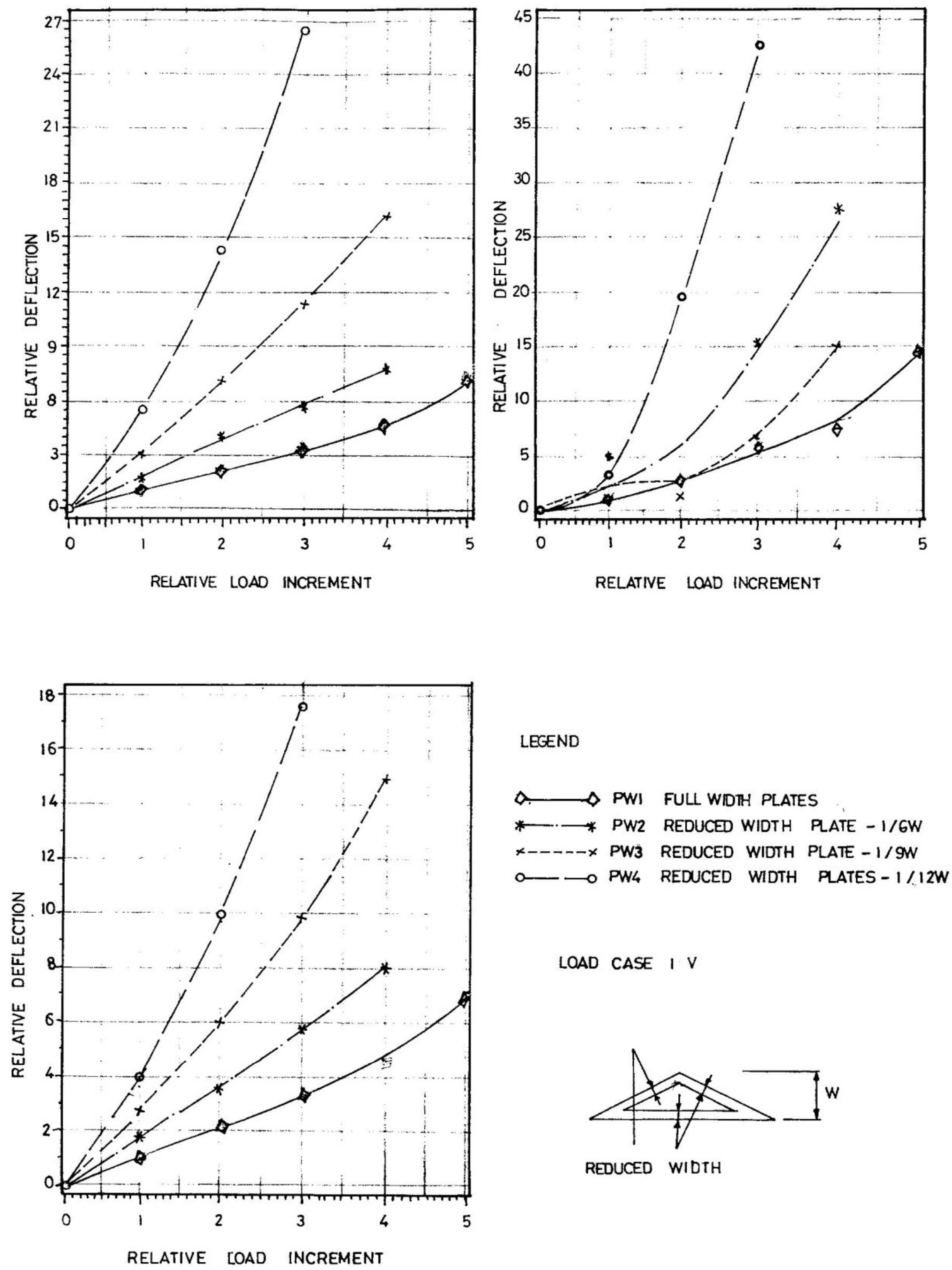

Fig. 9 Relative model deflections against relative load

3.2 STRESS

Rosette strain measurements were converted into principal stresses and maximum shear stresses using appropriate equations of the theory of elasticity. Rosette positions on the model are as illustrated in the developed plan shown in Fig. 10. The principal stresses obtained were generally of lower magnitude compared to the yield limit in tension of the model material, 5.2 N/mm^2. Relationships of stress against service load, at a selected rosette position at the crown of the model (position 26 on Fig. 10), are shown in Fig. 11[7]. The maximum minor principal stress obtained was 2.3 N/mm^2 compression, with a corresponding major principal stress of 0.1 N/mm^2 compression and maximum shear stress of 1.1 N/mm^2. The results also generally demonstrate that the greater the reduction in plate width the higher the stresses in the model. Deviations from the full-plate model increase as the magnitude of the load increases, indicating a diminishing capacity to sustain higher loads by models with reduced plate widths.

Fig. 12 shows the relative model surface stresses against relative load, corresponding to the results shown in Fig. 11. The first load increment and its corresponding stress on model PW1 have been taken as reference values in determining the relative quantities in the diagrams.

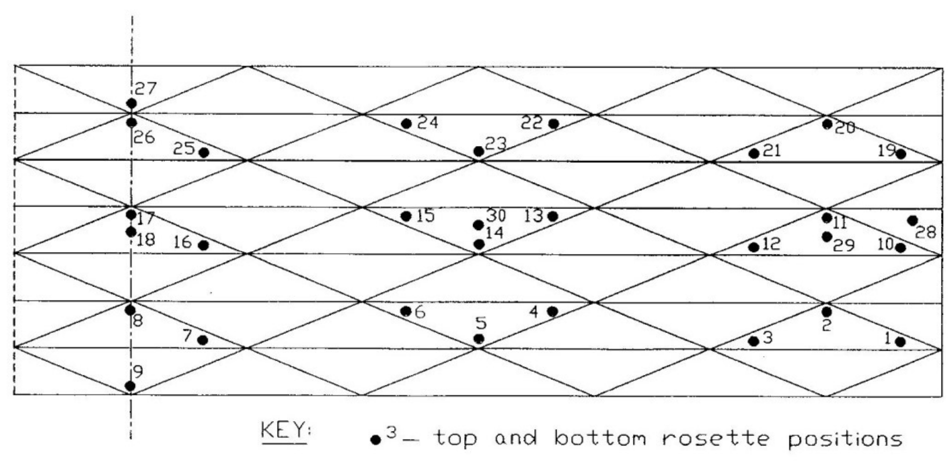

Fig. 10 Rosette strain gauge positions on model for determination of stress and strain[7]

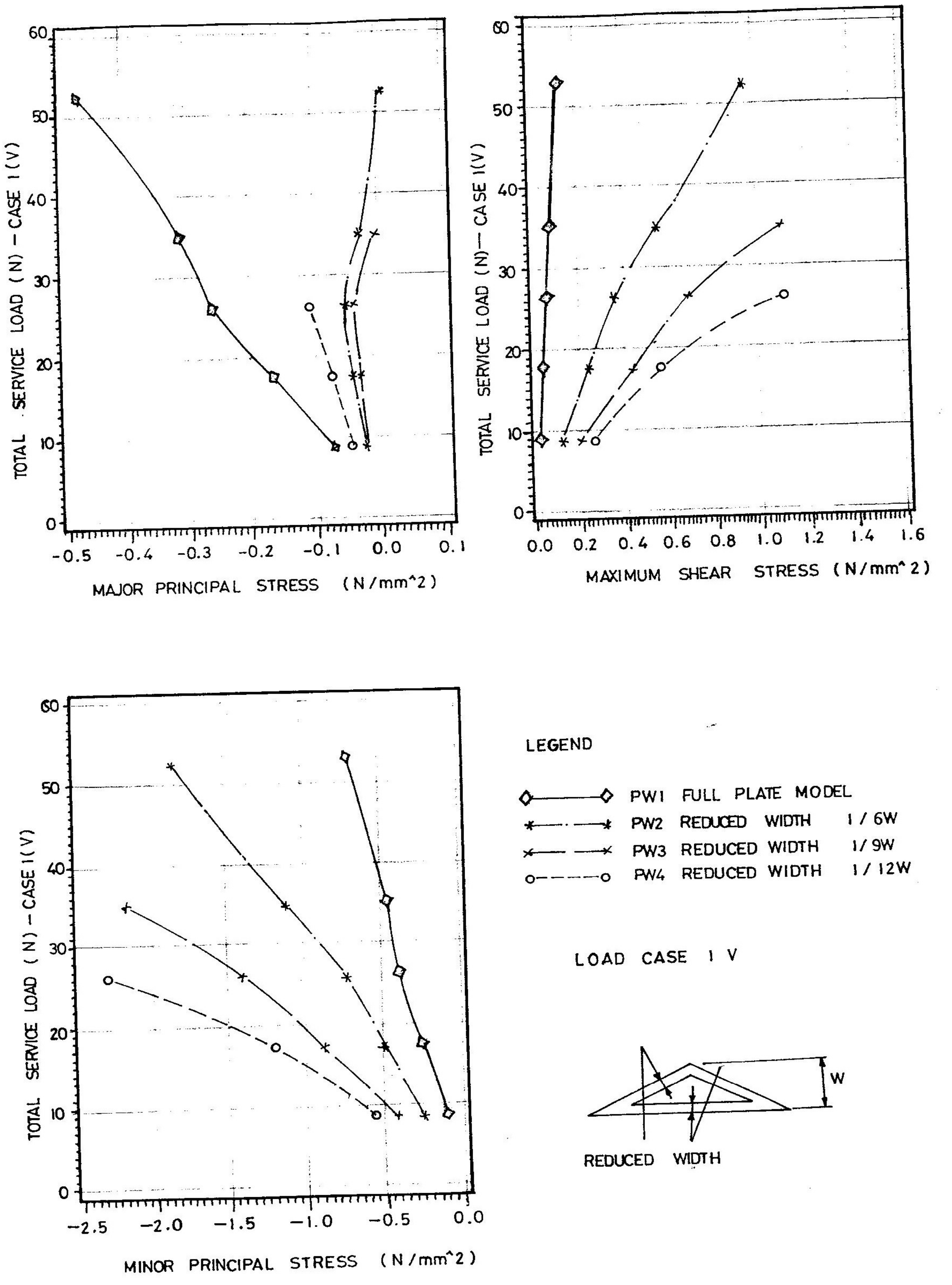

Fig. 11 Influence of plate width reduction on Surface Stress

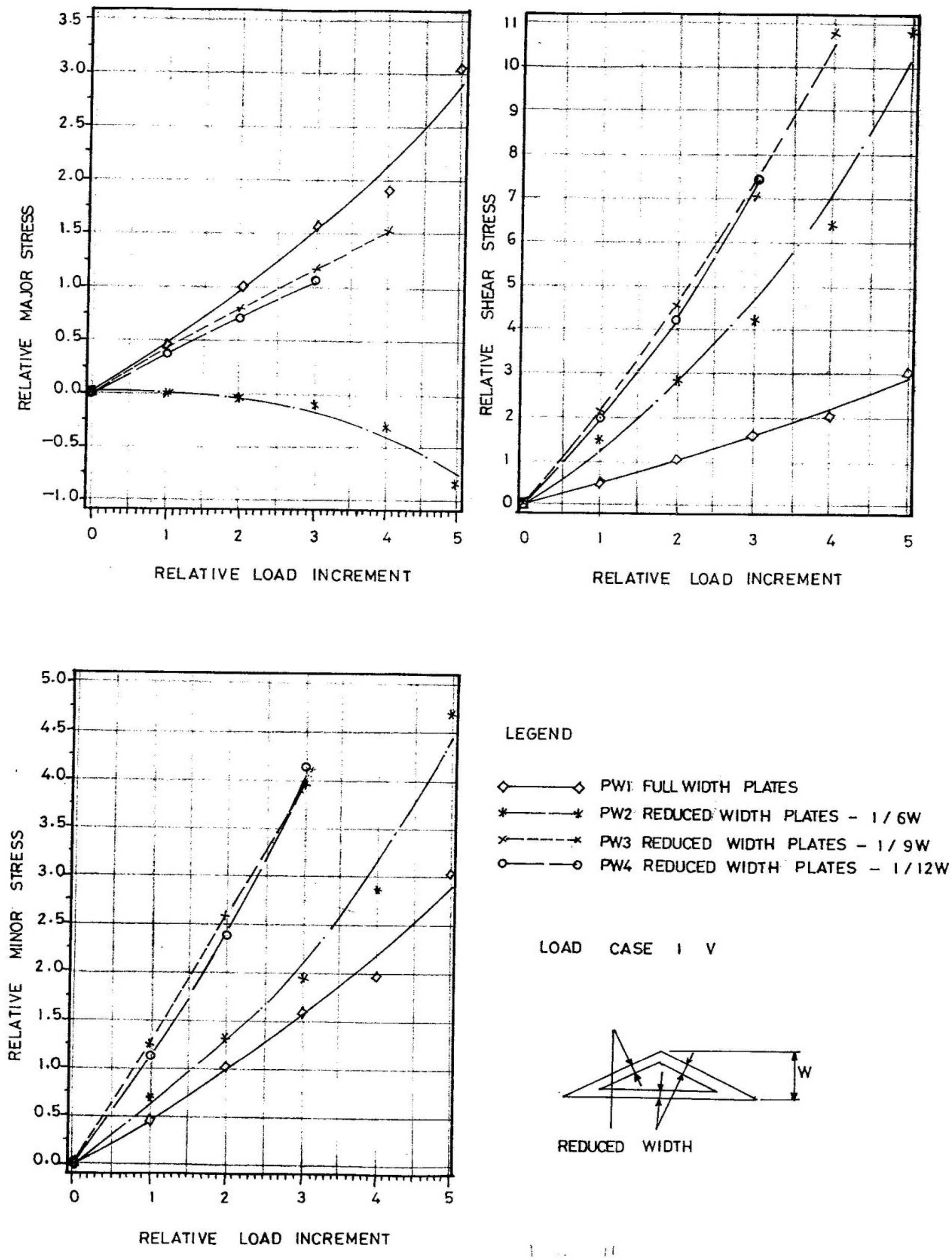

Fig. 12 Relative surface stress against relative load

4.0 DISCUSSION OF RESULTS

The results support Benjamin's[2] advice against physical reduction of plate width in order to achieve theoretical effective width. Similarly, Gilkie & Robak's[6] deformation studies on single skin hexagonal based pyramids showed that deflection properties of a physically reduced and complete pyramids were similar only up to a certain maximum level of stress, depending on the overall stiffness of the structure.

5.0 CONCLUSIONS

Physical reduction of plate width to approximate 'theoretical' effective width reduces the structural capacity of a triangularly folded plate barrel vault. However it is also found that a reduction of plate width to about 1/6th of full plate width gives a structure which is capable of sustaining relatively high loads.

It is projected that if a stiffer and stronger material was used for the skeletal structure better results would have been obtained even at significant reductions of plate width. This conclusion is supported by the low values of stress obtained at maximum load, indicating that the failure was largely due to serviceabilty limit being reached.

The obtained results also confirm suggestions made in earlier studies of the structure, that although the central part of a plate may not be effective in carrying the load, it cannot actually be removed, since it serves to stabilize the folds and helps to prevent torsional buckling at higher loads[2].

The results are useful when considering the design and construction of low-cost skeletal triangularly folded plate vaults, employing stronger materials at the folds and cheaper membrane materials to cover the central parts.

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Optimum Design for Spring with Orthogonal Design Method

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Abstract:

Orthogonal design method is a sort of direct optimum method, by which the optimal result may be obtained through simple computation and analysis. This method has gained increasingly extensive interest in scientific research, engineering design, and production management as well.

The orthogonal table is the basic instrument for carrying out optimum design with orthogonal design method. It is a mathematical table established on the basis of the idea of isostatic distribution. As various levels may appear in any column, it makes a partial experiment contain all levels of all factors; furthermore, all combinations of any two columns may appear, which makes any experiment between two factors become overall experiment. Though the arrangement of the orthogonal table is only for a partial experiment, nevertheless it may lead to understand the conditions of overall experiment.

While making optimum design with orthogonal table, it may use the design variables of the mathmatical model for optimum design for the factors of orthogonal table, and use the objective function f(X) for the experiment target. By means of orthogonal table for the arrangement of design plan, it is easy to determine an optimum design plan.

This article recommends the basic principles of using orthogonal design method to proceed with optimal design, and the process of how to use orthogonal design method for the optimization of spring.

From the theory and examples of this article, it may clearly understand the superiority of applying orthogonal design method to conduct optimum design. The optimum design can be obtained through simple computation. It is particularly suitable to conditions that the design variables are integer, or discrete variables.

1.0 Introduction

Orthogonal design method is a sort of direct optimum method, by which the optimum result may be obtained through simple computation and analysis. This is why the application of orthogonal design method has gained increasingly extensive interest in scientific research, engineering design, and production management as well.

2.0 The Basic Principles of Orthogonal Design Method For Optimum Design:

The orthogonal table is the basic instrument for carrying out optimum design with orthogonal design method. It is a mathematical table established on the basis of the idea of isostatic distribution. The general orthogonal table with equal levels can be written as $L_a(b^c)$, in which "L" indicates orthogonal table, "a" indicates the rows of orthogonal table, that is the number of experiments to be taken when experiments are arranged by orthogonal table; "c" indicates the columns of orthogonal table, that is the number of factors that the orthogonal table can arrange at most; "b" indicates the number of levels of orthogonal table in the arrangement of factors, that is the different values obtained by factors in experimental designs.

Let's take a most simple orthogonal table $L_4(2^3)$ (see Table 1) for observation. We can see:

Table 1 $L_4(2^3)$

Experiment No. \ Column No.	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

(1) Each level appears in any columns with equal times of appearance.

(2) All combinations of various levels appear in any two columns, and the joint arrangement is isostatic.

As various levels may appear in any column, it makes a partial experiment contain all levels of all factors; furthermore, all combinations of any two columns may appear, which makes any experiment between two factors become overall experiment. Though the arrangement of the orthogonal table is only for a partial experiment, nevertheless it may lead to understand the conditions of overall experiment.

While making optimum design with orthogonal table, it may use the design variables of the mathematical model for optimum design for the factors of orthogonal table, and use the objective function f(X) for the experiment target. By means of orthogonal table for the arrangement of design plan, it is easy to determine an optimum design plan.

3. 0 A practical example is shown below to introduce how to make optimum design for spring by using the method of orthogonal design.

Example: Try to design a cylinder spring with compressed spirals. Its deformed dimension $\lambda=16.59$mm. The maximum working temperature is 150℃, material requires chromium vanadium steel 50CrVA. The desired working life is 10^5 cycle index. After intensified treatment, its allowable shearing stress $[\tau]=404.9$MPa. The required rigidity of spring c=41N/mm. The axial pressure that the spring endures F=680.2N. Try to determine the wire diameter d, and medium diameter of the spring D, and number of effective coils n. It is requested to design, under the conditions to meet the strength and rigidity requirements, a structure plan in minimum weignt:

Solution: 3.1 Write out the optimum mathematical model of this example.

3.1.1. Design variables

Design variables are the wire diameter d, medium diameter of the spring D, and number of coils n.

$$\therefore X=[x_1, x_2, x_3]^T=[d, D, n]^T \quad (1)$$

3.1.2. Objective function

Taking the number of noneffective coils $N_1=1.8$. The weight target of the spring to be calculated by its volume is:

$$V=\frac{\pi_2}{4}d^2Dn+\frac{\pi_2}{4}d^2Dn_1 \quad (2)$$

$$\therefore f(X)=\frac{\pi^2}{4}d^2Dn+\frac{\pi^2}{4}d^2Dn_1=2.47x_1^2 x_2 x_3+4.44x_1^2 x_2 \quad (3)$$

3.1.3. Design Constraints

Basing on the strength requirement of the spring,

$$\tau=\frac{8k_1FD}{\pi d^3}\leqslant[\tau] \tag{4}$$

in which k_1 —— curvature coefficient.

$$k_1=\frac{4C-1}{4C-4}+\frac{0.615}{C}=\frac{1.6}{\left(\frac{D}{d}\right)^{0.14}} \tag{5}$$

C —— spring exponent, $C=\frac{D}{d}$

F —— Axial pressure endured by the spring, N.

From this computation we can have:

$$g_1(X)=\frac{8k_1FD}{\pi d^3[\tau]}-1=8\cdot\frac{1.6}{\left(\frac{D}{d}\right)^{0.14}}\cdot\frac{FD}{\pi d^3[\tau]}-1$$

$$=6.84x_1^{-2.86}x_2^{0.86}-1\leqslant 0 \tag{6}$$

Basing on the figidity requirement of the spring

$$n=\frac{Gd^4}{8cD^3}. \tag{7}$$

where, G—shearing elastic modulus of the spring material, and its value here is 8×10^4MPa

From this we can have

$$g_2(X)=\frac{Gd^4}{8cD^3n}-1=243.90x_1^4x_2^{-3}x_3^{-1}-1\leqslant 0 \tag{8}$$

According to experience,

$$4\leqslant\frac{D}{d}\leqslant 14 \tag{9}$$

From this, we can have

$$g_3(X)=4-\frac{x_2}{x_1}\leqslant 0 \tag{10}$$

$$g_4(X)=\frac{x_2}{x_1}-14\leqslant 0 \tag{11}$$

From the analysis shown above, it is known that the optimum mathematical model of this example is

$$\min f(X)=2.47x_1^2x_2x_3+4.44x_1^2x^2 \tag{12}$$

s. t. $$g_1(X)=6.84x_1^{-2.86}x_2^{0.86}-1\leqslant 0 \tag{13}$$

$$g_2(X)=243.90x_1^4x_2^{-3}x_3^{-1}-1\leqslant 0 \tag{14}$$

$$g_3(X)=4-\frac{x_2}{x_1}\leqslant 0 \tag{15}$$

$$g_4(X)=\frac{x_2}{x_1}-14\leqslant 0 \tag{16}$$

$$x_1, x_2, x_3>0$$

3.2 Select an appropriate orthogonal table for the preparation of orthogonal design plan.

As the design variables, i.e. factors have been determined, the number of factors' levels can be determined by experience basing on the intention of the subject. If no experience in this area, the original value may be given first, and then try to find another level values according to geometric series or arithmatic series. The selected orthogonal table should be able to contain all factors needed. A sheet of orthogonal table is only use for one round of design, or it may have several sheets of orthogonal tables combined into one, if necessary. When a round of orthogonal tables has completed the selection of optimum goal, it is also necessary to decide whether another round is necessary to be carried out according to the result of this round.

This example is composed of three design variables, and three numerical values are taken from each design variable. Thus a table of factor levels is formed. (see table 2)

The selected orthogonal table should be able to contain all factors and all factor levels. It can be seen from this point that it is appropriate to select orthogonal table $L_9(3^4)$ (See table 3)

Table 2 Table of factor levels

level \ factor	d	D	n
1	3	12	5
2	5	20	10
3	10	30	20

Table 3

ordinal numbers \ Factors	Design variables			Objective functions f(X)	Constraints g(X)
	x_1(d)	x_2(D)	x_3(n)		
1	①3	①12	①5	1813.32	Constraints violated
2	①3	②20	②10	5245.2	Contraints violated
3	①3	③30	③20	14536.8	Constraints violated
4	②5	①12	②10	8742	Constraints violated
5	②5	②20	③20	26920	Constraints Satisfied
6	②5	③30	①5	12592.5	Constraints violated
7	③10	①12	③20	64608	Constraints violated
8	③10	②20	①5	33580	Constraints violated
9	③10	③30	②10	87420	Constraints violated
Y_{j1}	21595.32	75163.32	47985.82		
Y_{j2}	48254.5	65745.2	101407.2		
Y_{j3}	185608	114549.3	106064.8		
R_j	164013	48804.1	53421.38		
good level	d_1	D_2	n_1		
Primary and secondary factors	d, n, D				
Optimum combination	$d_1 D_2 n_1$				

A factor can be arranged in each column of the orthogonal table, and d, D. n are arranged respectively in the first three columns of $L_9(3^4)$, and no factor can be put in column 4, which has no effect in orthogonal design, thereby it may be eliminated from the table. Then the different figures of all columns in the orthogonal table can be changed into corresponding levels of the homologous factors.

Combine it to this example, the first column is occupied by d, and 3(mm) are written behind the three numerals "1" of the first column, that is level 1 corresponds to factor d; and 5 (mm)—that is level 2 of d, are written behind the three numerals "2" of the first column: and 10(mm) that is the level 3 of d, are written behind the three numerals "3"of the first column. The remaining two columns are anagolous. And in this way an orthogonal design plan is formed. Every horrizontal line of the table represents a design plan. If the value of the objective function of every design plan can be figured out and the constraints are examined whether satisfied or not, the result shown in Table 3 is obtained.

3.3 Analysis of the design result.

It is apparent from table 3 that the value of objective function of the 5th design plan is in the middle, however, it may satisfy all the constraints, which is the better combination.

$X=[x_1, x_2, x_3]^T=[d, D, n]^T=[5, 20, 20]^T$, $f(X)=26920$

Whether this design plan is optimum combination or not, it can be simply (or through audio-visual) analyzed by using the method of maximum difference. In table 3, Y_{jk} is the sum of objective functions of design plans corresponding to j factor of K level. The good level combination of various factors is the optimum combination. The reason is the value of objective function being "the less the better". It can be seen from table 3 that the optimum combination of this example would be $d_1 D_2 n_1$. R_j in table 3 is the maximum difference of factor j. Its computing formula is:

$$R_j=\max[Y_{j1}, Y_{j2}, Y_{j3}]-\min[Y_{j1}, Y_{j2}, Y_{j3}] \quad (17)$$

The bigger the R_j, means the bigger the influence of the design variable on the value of the objective functions. The primary and secondary design variables can be judged from this. It is clearly seen from table 3 that the primary and secondary sequence of factors in this example is d, n, D. The better combination directly obtained from the table is $d_2 D_2 n_3$. It shows that there probably exists another best design plan. Since d and n are all discrete variables, if a smaller value is taken (if d=4.5mm, n=19) though the values of objective functions were decreased, nevertheless they can not satisfy the constraints. Therefore the optimum solution is still like this:

$X^*=[d, D, n]^T=[5, 20, 20]^T$, $f(X^*)=26920$

If this example can be determined by using the method of geometric programme, it may obtain:

$x_1^*=d=4.89mm$

$x_2^*=D=20.01mm$

$x_3^*=n=14.57$

According to related standards and practical conditions, 5 mm can be used for the diagram of the spring wire, medium diameter of the spring can be 20 mm round, and the number of effective coils can be 15 round. Its value of the objective function can be 20745. If compared with this example ,it is obvious that the design plan is comparatively better. If d=5mm, D=20mm, n=15, suppose these figures are used in the original optimum mathematical model, however, they can hardly meet the constraints. This is why the optimum solution determined from this example is still the optimum solution.

4.0 **Conclusion:**

4.1 The application of orthogonal design method to handle optimal design problems is a direct optimization method. The overall conclusion can be obtained through a little computation.

4.2 It can be seen from the result of this article the superiority of applying optimum design with orthogonal design method, and it is particularly suitable to conditions that the design variables are integer, or discrete variables.

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