A CHECK PARAMETER METHOD FOR USE IN DESIGNING RELAYS

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ABSTRACT

Low quality magnetic steel available locally has been successfully used as core material for a relay. New design principles as well as construction procedure used in development of a prototype are presented. Then the new approach is assessed and found to deliver acceptable accuracy.

INTRODUCTION

Design methods presently in use for relays are cumbersome and labour consuming. In most cases they require the use of a huge amount of empirical formulae and graphs\textsuperscript{[1,2]}. On the other hand, only computers with big storage capability have proved suitable for such procedures. The new approach, proposed here, shortens considerably the design procedure by calculating only few vital check parameters. The quantity of mathematical expressions and graphs used for the process is reduced. Then the validity of the method is evaluated.

The procedure that is suggested here is successfully used for constructing the main element in the relay and is also very simple and efficient.

CHECK PARAMETERS

The check parameters are five, namely relative magnetic resistance, inductance, magnetic energies, torque and pulling force. The computation method is described here below.
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The Relay winding

The relative magnetic resistance is obtained using the expression:

\[ R_m = \frac{IR_{av} + qR_B}{1 + \frac{R_B}{IR_{av}}(q - 1)} = \frac{3(IR_{av} + qR_B)}{3 + R_Bgl} \]  \hspace{1cm} (1)

Where \( R_{av} \) - average magnetic resistance per unit length of the relay core and base; \( R_B \) - magnetic resistance of the relay airgap taking into account the fastenings attaching the core to the relay base; \( l \) - length of core in metres; \( g \) - magnetic admittance of the leakage flux within a unit length (1 metre) of the magnetic circuit and \( q \) is represented by:

\[ R_m = \frac{lR_{av} + qR_B}{l + \frac{R_B}{lR_{av}}(q - 1)} = \frac{3(lR_{av} + qR_B)}{3 + R_Bgl} \] \hspace{1cm} (2)

Neglecting the resistance of the soft iron portion results into:

\[ q = \frac{l\sqrt{gR_{av}}}{\tanh l\sqrt{gR_{av}}} \] \hspace{1cm} (3)

Where \( L'_o \) is the static inductance (when resistance of soft iron portion is neglected.

If \( R_{ov} \) the magnetic resistance of the joint between the core and the casing at the base of the former, is taken into account the following expression is obtained:

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\[ L_o = \frac{(2 + R_agl)(3 + R_BgI)N^2}{6[(R_{av} + R_BRBg)l + q(R_o + R_B)]} = K_o'N^2 \]  

(4)

Neglecting conductance caused by flux leakages \( g = 0 \) results into:

\[ L_o'' = \frac{N^2}{lR_{av} + R_o + R_i} = K_o''N^2 \]  

(5)

![Figure 1 - A type of \( K_o \) curves used for designing relays](image)

The value \( K_o \) depends upon the configuration of the relay, the size of the space inbetween the soft irons and ampere turns. Estimation of \( K_o \) using known equations may prove to be time consuming. However, it is somewhat easier to obtain the value experimentally. Figure 1 shows a series of curves for different sizes of space obtained experimentally, for the low quality magnetic steel. Inductance was measured using the ballistic method and the Maxwell Bridge at constant current.
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From the curves it is seen that the relay inductance is maximum within the range from 70 to 200A. At ampere turns greater than 300A inductance is independent of values of space in between the soft-irons.

Inductance of a winding without a soft iron core may be calculated with the help of the following equation (cylindrical coils with \( O < D < p \))

\[
L = 10.1N^2D^3\sqrt{(\frac{D}{p})^2}10^{-9} \, H
\]  

(6)

where \( D \) - average diameter of the coil; \( p \) - perimeter of the cross-sectional area of the conductors, cm.

Magnetic Energy

Magnetic energy stored before movement of the armature:

\[
W_1 = \int_{0}^{\psi_1} id\psi = Area_0a_1b_1
\]

(7)

Magnetic energy obtained by the system within the motion time of the armature:

\[
W_2 = \int_{0}^{\psi_2} id\psi = Area_1a_1b_1b_2a_2
\]

(8)

Magnetic energy stored after movement of the armature:

\[\text{Uhandisi Journal, Vol. 17 No. 2, December, 1993}\]
\[ W_3 = \int_0^{w_2} id\psi = \text{Area}a_2b_2 \]  

(9)

Magnetic energy consumed during motion of the armature is given by:

\[ W_m = \frac{1}{2} I\psi_1 + I(\psi_2 - \psi_1) - \frac{1}{2} I(\psi_2 - \psi_1) \]  

(10)

In case of small linkages \( \psi = \phi N \) the following expression is obtained:

\[ F = -\frac{1}{2} I \frac{d\psi}{d\delta} = -\frac{1}{2} IN \frac{d\Phi}{d\delta} = -\frac{(IN)^2}{2} \frac{dG_m}{d\delta} \]  

(11)

Torque

Torque delivered by the armature then becomes:

\[ T_a = \frac{(IN)^2}{2} = \frac{dG_m}{d\alpha} \]  

(12)

The total magnetic conductance of the relay:

\[ G_m = \frac{1}{G_\delta} + \frac{1}{G_\pi} \quad \text{and} \quad \frac{dG_m}{d\delta} = \frac{G_m^2}{G_\delta} \frac{dG_\delta}{d\delta} + \frac{G_m^2}{G_\pi} \frac{dG_\pi}{d\delta} \]  

(13)

where \( G_\delta \) - total magnetic conductance of the airgap which is dependent on rotational angle of armature, \( G_\pi \) - magnetic conductance of the other remaining parts of the magnetic system (core, base, armature and fastenings)
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associated with system) not dependent on rotational angle of the armature.

Hence:

\[ F = -\frac{(IN)^2 G_m^2}{2G_\delta^2} \frac{dG_\delta}{d\delta} \] \hspace{1cm} (14)

Then torque delivered becomes:

\[ T_a = -\frac{(IN)^2}{2} \frac{dG_m}{d\alpha} = -\frac{(IN)^2 G_m^2}{2G_\delta^2} \frac{dG_\delta}{d\alpha} \] \hspace{1cm} (15)

If assume that during the motion time of the armature, flux does not change and the magnetic characteristic of the relay is a straight line (figure 2) then neglecting leakages, the following expression is obtained:

\[ W_m = \frac{1}{2} I_1 \psi - \frac{1}{2} I_2 \psi = \frac{1}{2} \psi (I_1 - I_2) = \frac{1}{2} \Phi^2 (R_m1 - R_m2) \] \hspace{1cm} (16)

In this case, pulling force has the following expression:

\[ F = \frac{\Phi^2}{2} \frac{dR_m}{d\delta} \] \hspace{1cm} (17)

and torque delivered by the armature is given by:

\[ T_a = \frac{\Phi^2}{2} \frac{dR_m}{d\delta} \] \hspace{1cm} (18)
Pulling Force

Total magnetic conductance of the relay airgap can be expressed by the following equation:

\[ G_\delta = G + G_1 + G_2 \text{ and } \frac{dG_\delta}{d\alpha} = \frac{dG}{d\alpha} + \frac{dG_1}{d\alpha} + \frac{dG_2}{d\alpha} \]  \hspace{1cm} (19)

where

\[ \frac{dG}{d\alpha} = -\frac{2\pi\mu_o}{a^2} (c_1 - \sqrt{c_1^2 - r^2}) \]  \hspace{1cm} (20)

\[ \frac{dG_1}{d\alpha} = \frac{\pi^3\mu_o c_1}{2(\frac{\pi}{2} + 1)^3 \cos^2 \alpha} \]  \hspace{1cm} (21)
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Substituting the value of \((IN)^2\) by the value \(\Phi^2/G^2\) the following expression is obtained:

\[
T_a = \frac{\Phi^2}{4\pi\mu_o(c_1 - \sqrt{c_1^2 - r^2})}
\]

For small rotational angles of the armature \(\alpha\) is considered approximately equal to \(\delta/c_1[3]\) and hence:

\[
G = \frac{2\pi c_1}{\delta} \mu_o(c_1 - \sqrt{c_1^2 - r^2})
\]

\[
\frac{dG}{d\delta} = -\frac{2\pi c_1}{\delta^2} \mu_o(c_1 - \sqrt{c_1^2 - r^2})
\]

Pulling force acting on the armature (directly opposite the centre of the core) is:

\[
F = \frac{(IN)^2}{2} \frac{2\pi c_1}{\delta^2} \mu_o(c_1 - \sqrt{c_1^2 - r^2}) = \frac{\Phi^2}{4\pi c_1 \mu_o(c_1 - \sqrt{c_1^2 - r^2})}
\]

CONSTRUCTION OF THE MAIN ELEMENT

The magnetic system is single-coiled with a c-shaped core. The core is a series of plates from low quality steel[4]. Its cross-section is a complicated configuration of a number of rectangular shapes inscribed in a circle. The circle is, in fact, the cross-section of the inner part of a cylindrical winding engulfing the core. The armature, which also forms part of the magnetic system, is made from the same steel as that of the core.

RESULTS

The present design methods using a computer was done so as to be able to compare with results obtained using the method presented here. Details of the results obtained are shown in Table 1. Check parameters namely relative magnetic resistance, inductance, magnetic energies, torque and
pulling force were involved in the calculations. The average deviations as compared to values obtained through existing methods were found to be 0.80%, 1.12%, 1.40%, 1.60% and 1.70% respectively. Moreover, the time needed for the calculations was found to have reduced considerably because of a huge reduction in the use of empirical formulae and graphs.

<table>
<thead>
<tr>
<th>Table 1 - Results of calculations to various relays based on both methods.</th>
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<td>Relay</td>
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<tr>
<td>Deviation (%)</td>
</tr>
<tr>
<td>Average deviation (%)</td>
</tr>
</tbody>
</table>

Relay 1 - Small size relay (I_{op} = 3A)
Relay 2 - Medium size relay (I_{op} = 15A)
Relay 3 - Large size relay (I_{op} = 28A)

**DISCUSSION OF RESULTS**

Existing methods for designing relays are based on a series of assumptions. This is due to the fact that the process taking place during operation of the relay is complicated because of the non-linear relationship between the magnetizing current and flux, the non-uniform distribution of the flux along the cross-section and length of the core, the effect of eddy currents and leakages, and the non-uniform movement of the armature during operation of the relay. The method presented and discussed in this paper avoids some of these complications through the use of a simpli-
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fied model. The deviations of the check parameter values as compared to values obtained through existing methods were found to be within an acceptable range for all the relays sampled.

CONCLUSION

The method proposed in this paper has advantages in that it reduces the amount of time for calculations and releases storage capacity of computers. The answers obtained were also found to be within accuracy limits. The method may hence prove useful to relay designers.

REFERENCES