MODEL FOR COMPLETE DRAWDOWN OF FLOATING SOLIDS IN STIRRED TANK REACTORS

J.P. Mmbaga[1] and P.N. Armenante[2]

[1] Chemical Engineering Dept., P.O. Box 35131 Dar es Salaam Tanzania
[2] Department of Chemical Engineering, Chemistry and Environmental Science, New Jersey Institute of Technology, Newark NJ 07102 U.S.A.

ABSTRACT

A Semi-empirical model has been developed on the basis of Kolmogoroff's theory of isotropic turbulence to determine the minimum drawdown speed for floating solids in a mechanically agitated vessel. An adjustable parameter in the model is found to be a function of impeller type and position in a vessel. A simplified representation of macroscopic flow pattern, representing the circulation flow in a vessel depending on impeller pumping characteristic is developed to account for recirculation pattern.

INTRODUCTION

Fluid mixing is one of the most important aspects in chemical processing, biochemical and allied industries. Petroleum refining, pharmaceuticals, pulp and paper, waste treatment, food processing and beverages are but a few of examples of such industries where fluid mixing principles are applied intensively.

Liquid-Solid Contacting

Liquid-solid contacting is probably one of the most common application class in mixing technology and the mechanically agitated vessel, sometimes referred as the stirred tank reactor, is the most frequently used to bring about excellent liquid-solid contact.

The suspension of settling solids has received a lot of attention from many investigators, e.g. Zwietering [1], Nienow [2], Oldshue [3], Kolar [4], Baldi [5], and many correlations are available in literature for the determination of the speed of agitation required to achieve a desired degree of suspension. On the other hand, a limited number of investigations have been made of mixing of floating solids, which is of particular interest in the fields of fermentation, sewage treatment, mineral processing, polymerization reactions. This paper reports an attempt to develop a model, based on Kolmogoroff's theory of isotropic turbulence to determine the minimum agitation requirements for floating solids.

Turbulent Flow and Kolmogoroffs Theory

Turbulent flow is characterized by chaotic unsteady movements of parts of a fluid. Such chaotic movements of any fluid element are complicated in nature and can only be described in terms of average fluctuations. The result of these movements is a superposition of a spectrum of velocity fluctuations on an overall mean flow. Large eddies produced in a turbulent flow corresponds to the large velocity fluctuations of low frequency and are of the size comparable to the physical dimensions of the flow system. Smaller eddies of higher frequency are produced by the interaction of the large eddics with slow moving streams [3].

A measure of intensity of turbulence in a given direction is given by velocity fluctuation. The instantaneous velocity in a given direction consists of the time averaged velocity at any point in fluid and the instantaneous fluctuation velocity. Since the fluctuation velocity can be positive or negative, it is convenient to express the amplitude of the fluctuation velocities as mean squares of the fluctuation velocities, and then take the root of the mean of the squares to get a root mean square fluctuation velocity, i.e which is always positive. When the mean squares of the fluctuation velocities in all directions are the same i.e., we have a condition of isotropic turbulence. Kolmogoroff's theory of homogeneous isotropy may be summarized as: "At sufficiently high Reynolds numbers there is a range of high wave-numbers where the turbulence is statistically in equilibrium and uniquely determined by the parameters. This state of equilibrium is universal" [6].

From dimensional reasoning [11], a Kolmogoroff length scale can be defined

$$\eta = (\frac{v^3}{\varepsilon})^{1/4} \tag{1}$$

and similarly a velocity scale can be defined

$$\upsilon = (\frac{\nu}{\varepsilon})^{1/4} \tag{2}$$

According to this theory, if distances and velocities are referred to these scales, a universal function β exists such that

$$\frac{u}{\upsilon} = \beta(\frac{d}{\eta}) \tag{3}$$

where u is the r.m.s. relative velocity between two points in the fluid at a distance d apart. At the high Reynolds numbers then, an "inertial sub-range" exists in which the viscous dissipation is not important. In this region,

$$u \approx (\varepsilon d)^{1/3} \qquad \eta << d << L \tag{4}$$

which is equivalent to saying

$$\beta(\frac{d}{\eta}) \approx (\frac{d}{\eta})^{1/3} \tag{5}$$

For very short distances, the viscous forces cannot be neglected and the velocity gradient is constant, i.e.

$$\beta(\frac{d}{\eta}) \approx \frac{d}{\eta} \tag{6}$$

and so

$$u \approx d(\frac{\varepsilon}{\nu})^{1/2} \qquad d << \eta \tag{7}$$

MODEL DEVELOPMENT

Consider a particle floating on a liquid surface. This particle may be fully or partially immersed shown in Fig.1. The forces acting to keep the particle in its position are its weight, $mg = \rho_s V_s g$ and the upthrust, $F_B = (V_l^{'})\rho_l g$

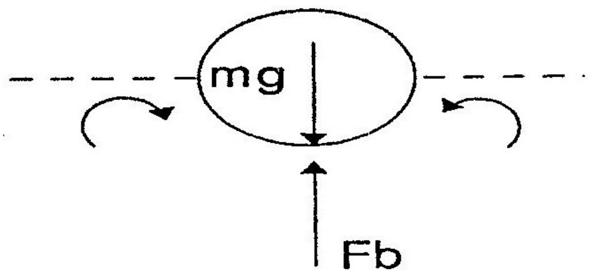

Fig. 1: Forces acting on a particle

In order to draw-down and suspend the particle, force must be exerted by the turbulent eddy to move the particle a certain distance which is sufficient for it to be entraped and maintained in circulation by the large circulating eddies in the main flow. Assuming that the flow conditions in the vessel are such that sufficiently large Reynolds numbers have been reached, we can estimate the force as:

$$F_E = \tau A^{'} \tag{8}$$

where A' is the area of the particle which is exposed to the eddy and τ is the associated shear stress. The fluctuating shear stress in isotropic turbulence may be formulated as [7]:

$$\tau = \rho_l . u^{'2} \tag{9}$$

and using Kolmogoroff's equation (4) with additional assumption that the distance d is of the order of the particle size and that the fluctuating velocity is proportional to the r.m.s. relative velocity may be approximated as:

$$u' \alpha (\varepsilon . d_p)^{1/3} \tag{10}$$

The mean energy dissipation rate in an agitated vessel is given by [4]:

$$\varepsilon = \frac{P}{\rho_l V} \tag{11}$$

and the power drawn by an impeller is given by

$$P = P_o \rho_l N^3 D^5 \tag{12}$$

combining (11) and (12), we can thus express ε as

$$\varepsilon = \frac{P_o N^3 D^5}{V} \tag{13}$$

whence equation (10) becomes

$$u' \alpha (\frac{P_o N^3 D^5 d_p}{V})^{1/3} \tag{14}$$

For a cylindrical vessel filled to height H, V = $\pi T^2 H / 4$
thus

$$u' \alpha (\frac{P_o N^3 D^5 d_p}{T^2 H})^{1/3} \tag{15}$$

from equation (9)

$$\tau \ \alpha \rho_l (\frac{P_o N^3 D^5 d_p}{T^2 H})^{2/3} \tag{16}$$

using this with equation (8)

$$F_E \alpha \rho_l (\frac{P_o N^3 D^5 d_p}{T^2 H})^{2/3} A' \quad (17)$$

or

$$F_E = \Phi' \rho_l (\frac{P_o N^3 D^5 d_p}{T^2 H})^{2/3} A' \quad (18)$$

where Φ' is a constant dependent on impeller type and position only.

For the eddy to be able to draw-down the particle, the eddy force must be greater than the net force due to gravity and upthrust or

$$\Phi' \rho_l (\frac{P_o N^3 D^5 d_p}{T^2 H})^{2/3} A' > (\rho_l V_l' g - \rho_s V_s g) \quad (19)$$

Now, for conditions where the particle will be just suspended we can replace N .by N_{jd} and use equality sign to get

$$\Phi' \rho_l (\frac{P_o N_{jd}^3 D^5 d_p}{T^2 H})^{2/3} A' = (\rho_l V_l' g - \rho_s V_s g) \quad (20)$$

which gives on rearrangement the critical speed as

$$N_{jd} = \Phi (\frac{(\rho_l V_l' / V_s - \rho_s)}{\rho_l})^{1/2} (\frac{V_s}{A'})^{1/2} (\frac{1}{P_o})^{1/3} \frac{(T^2 H)^{1/3}}{D^{5/3} d_p^{1/3}} \quad (21)$$

For a totally immersed spherical particle $A' = 4\pi R^2$ and

$V_s = \frac{4}{3}\pi R^3 = V'$ which gives $\frac{A'}{V_s} = \frac{6}{d_p}$ and $\frac{V'}{V} = 1$ respectively, and equation (21) simplifies to:

$$N_{jd} = \Phi(\frac{\Delta\rho g}{\rho_l})^{1/2} P_o^{-1/3} (T^2 H)^{-5/3} d_p^{1/6} \qquad (22)$$

EXPERIMENTAL

Model for Φ and Proposed Drawdown Mechanism

In a recent publication[8], Armenante and co-workers showed that the coefficients in the proposed equation do indeed correlate with experimental data to an appreciable extent, and also that there are similarities, as well as differences with settling solids systems. Since equation (22) was derived based on turbulence hypothesis, the aim of the present data analysis is to include circulation hypothesis in the equation by modeling the pseudo-constant Φ.

From experimental and theoretical observations, the pseudo-constant has been found to depend on impeller clearance ratio, and the impeller flow characteristics. It is proposed to relate the pseudo- constant with the distance covered by the largest circulating eddies, x' using a power function

$$\Phi = k_1 x^{k_2} \qquad (23)$$

where x is the macroscopic flow distance normalized by impeller diameter, and k_1 and. k_2 are constants to be determined experimentally.

With reference to Figs. 2a and 2b, the macroscopic distance can be approximated, depending on the type of impeller pumping direction and resulting flow pattern as:

for closed (DT) or open (FBT) disk blade turbine (radial flow)

$$x' = C + T - D \qquad (24)$$

pitched blade turbine pumping upwards (PBT_{DP}) (assuming axial flow only).

$$x' = C + \frac{T}{2} \qquad (25)$$

pitched blade turbine pumping downwards...(assuming axial flow only)

$$x' = 3T - C \qquad (26)$$

and the path may be normalized by impeller diameter to give a dimensionless macroscopic flow function $x = \frac{x'}{D}$

From experimental observation, the pitched blade turbine has been found to exhibit a mixed flow pattern. This is seen in the incipient complete drawdown speed as shown in Fig.2d and 2e. For this matter, the flow distance is modified to account for the pattern as shown in Fig.2f-2j. For a given pumping direction, the pitched blade turbine (PBT) has been observed to show a change in flow pattern [9] at a clearance of C/T=0.71 when pumping downwards and C/T = 0.29 when pumping upwards. Thus the modified macroscopic flow path function, $\bar{x}$ to account for for observed changes as defined in Table 1.

Table 1. Macroscopic Flow Path Functions}

Impeller type	Macroscopic flow path function
Disk turbine	$x = C/D + T/D - 1$
Flat blade turbine	$x = C/D + T/D - 1$
Pitched blade turbine (UP)	$x = C/(2D) + T/2D$
Pitched blade turbine (DP)	$x = 3T/D - C/D$
Disk turbine	$\bar{x} = x$
Flat blade turbine	$\bar{x} = x$
Pitched blade turbine (UP)	$\bar{x} = \sqrt{2}(T/(2D) - 1/4) + (C/D - T/(2D) - 1/4) + T/(2D)$ for $C/T > 0.29$
	$\bar{x} = \sqrt{2}C/D + (T/(2D) - 1/4)$ for $C/T < 0.29$
Pitched blade turbine (DP)	$\bar{x} = \sqrt{2}(T/2D - C/D) + (2T/D - 1/4 - C/D)$ for $C/T > 0.71$
	$\bar{x} = \sqrt{2}(T/(2D) - 1/4) + (T/2 + C/D) - 1/4)$ for $C/T < 0.71$

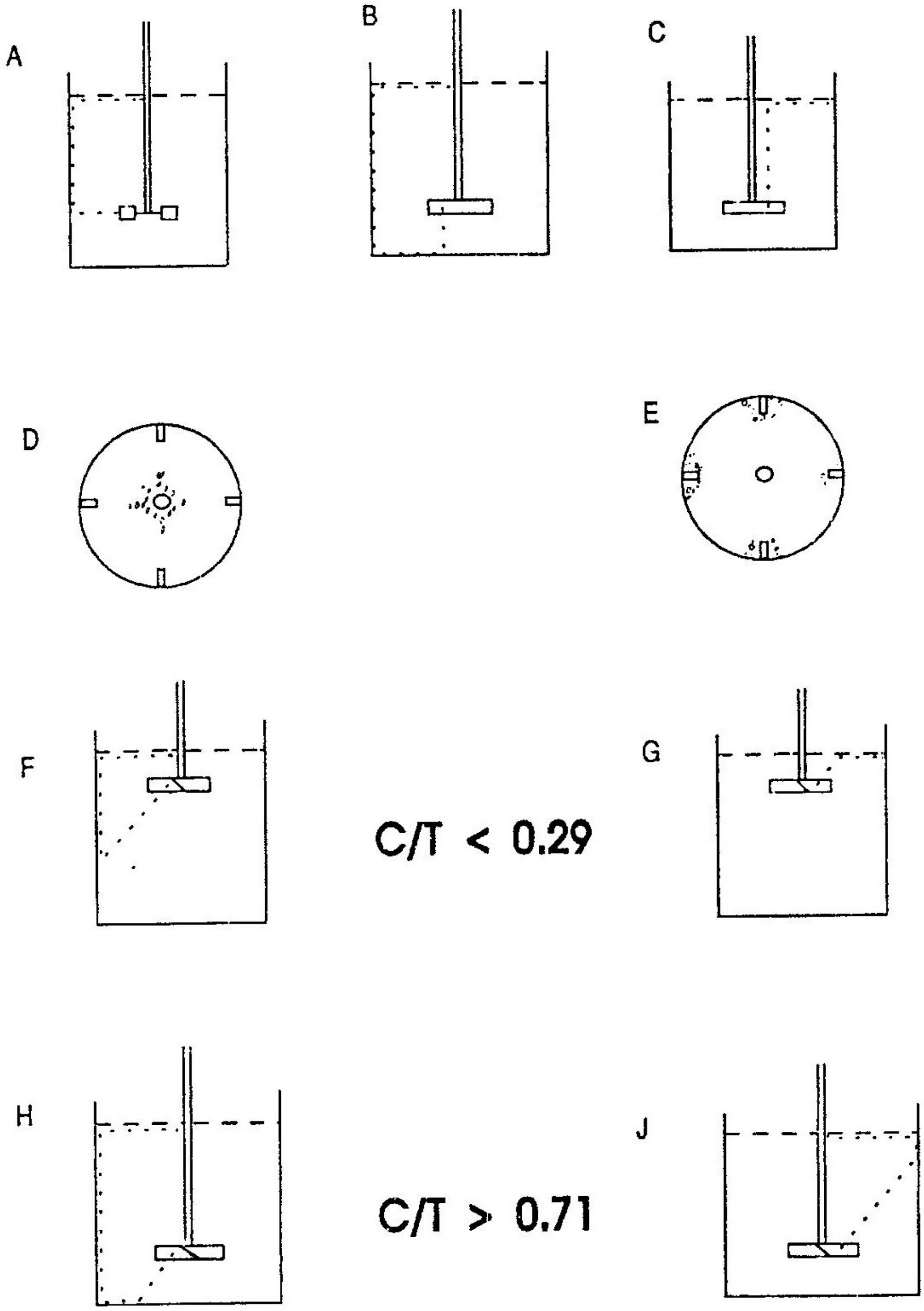

Fig. 2: Macroscopic flow path function - schematic

The proposed relationships were then tested by running experiments using different floating solids using setup shown in Fig.3 below. Details of experiment are reported at length elsewhere [8,9].

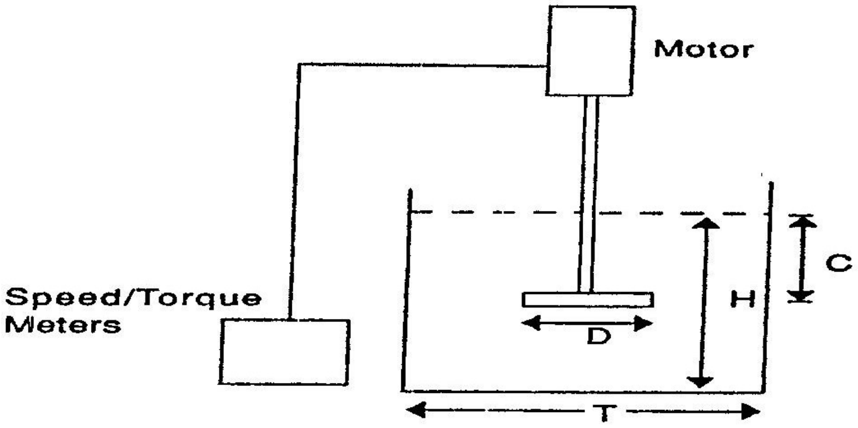

Fig. 3. Schematic of experimental setup

RESULTS AND DISCUSSION

Table 2 shows the constants k_1 and k_2 for the different impellers tested for floating solids drawdown. It is interesting to note that the data for disk turbine, pitched blade turbine (pumping upwards) and flat blade fall within a region of slope 1.4 to 2.1. On the other hand, data for.PBT_{DP} lies on a line of negative slope. However, as shown in Fig.4a, or more explicitly in Fig.4b, when PBT_{DP} changes pattern at higher C/T values, it actually aproaches the DT pumping pattern.

Table 2 Values for k_1 and k_2

Impeller	k_1	k_2
Disk turbine, *DT*	7.4	1.45
Flat blade turbine, *FBT*	4	2.1
Pitched blade turbine, PBT_{UP}	5.6	1.97
Pitched blade turbine, PBT_{DP}	110	-0.33

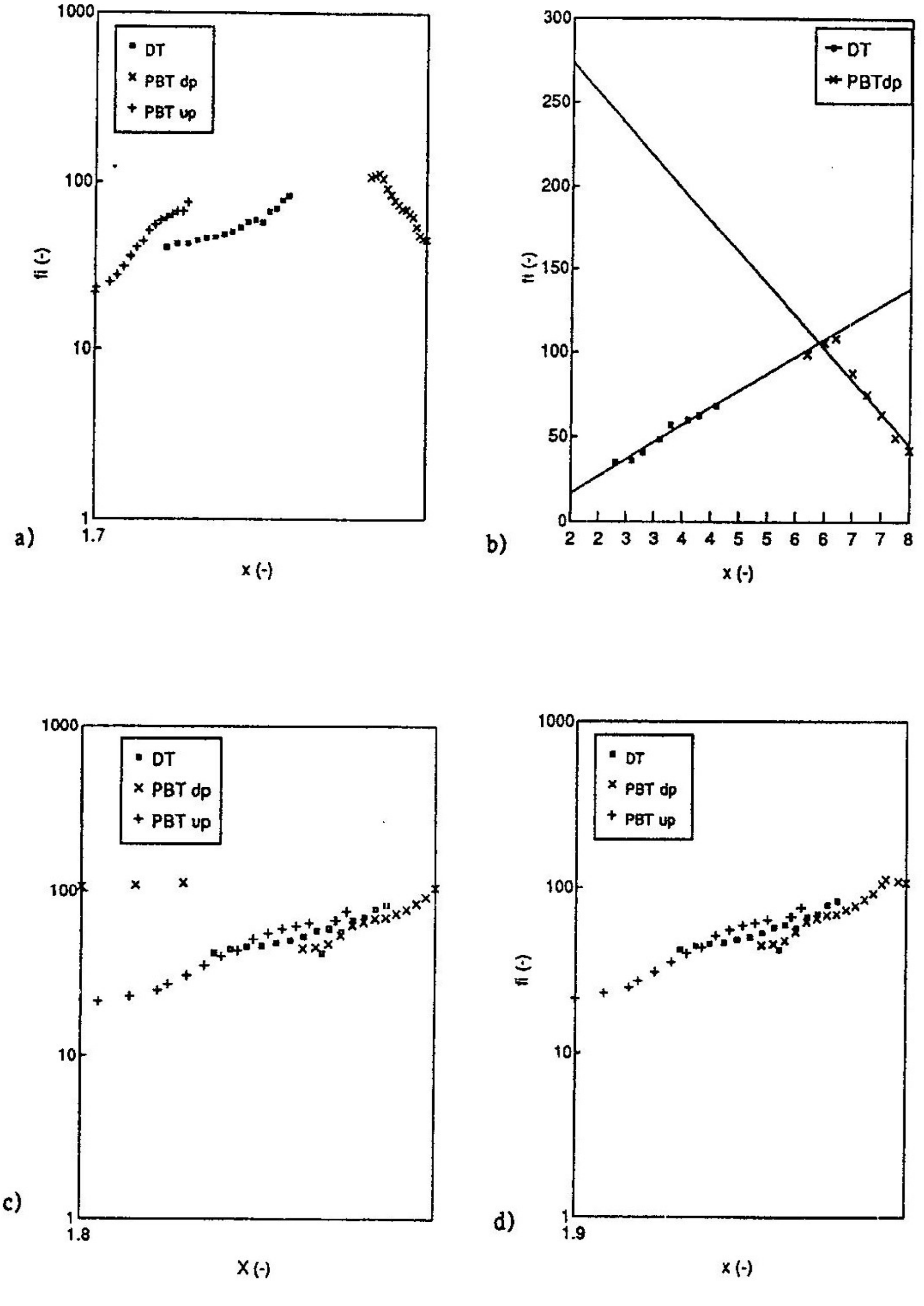

Fig. 4: Plots of f1nand flow path function

Figs.4c and 4d shows that the modified flow path function, $\bar{x}$ does indeed account for the observed discrepancies and reduces the data for all impellers as a function of $\bar{x}$. In Fig.4c, the modification to x is done according to theory and some data for low x are thrown off the rest, but when the modification is done to x only at experimentally observed points, uniformity is conformed (Fig. 4d). Figs. 5 and 6 show that there is a considerable change in minimum drawdown speed and the corresponding power consumption occurs as the clearance ratio passes the C/T=0.71 point for PBT pumping downwards. However, the changes are not dercenible when the impeller is pumping upwards.

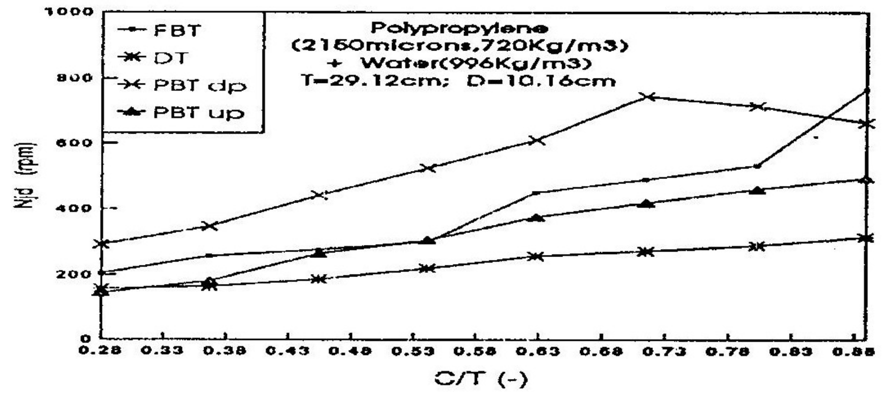

Fig. 5: Minimum drawdown speed for increasing C/T ratios

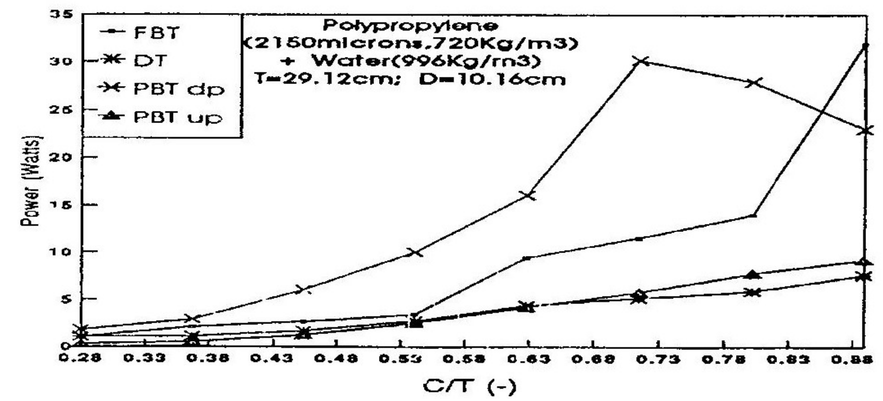

Fig.6: Power consumption at Njd for varying C/T ratios

Figs. 7 and 8 shows values in different systems tested and all data is reduced into two close regions. One region encompasses data for all impellers types used except for PBT_{DP} is shifted in its own zone.

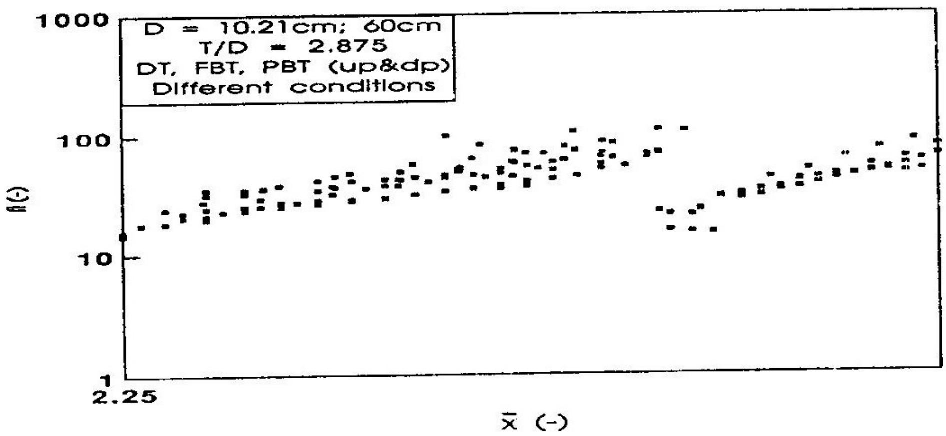

Fig. 7: Plot of fi versus flow path function

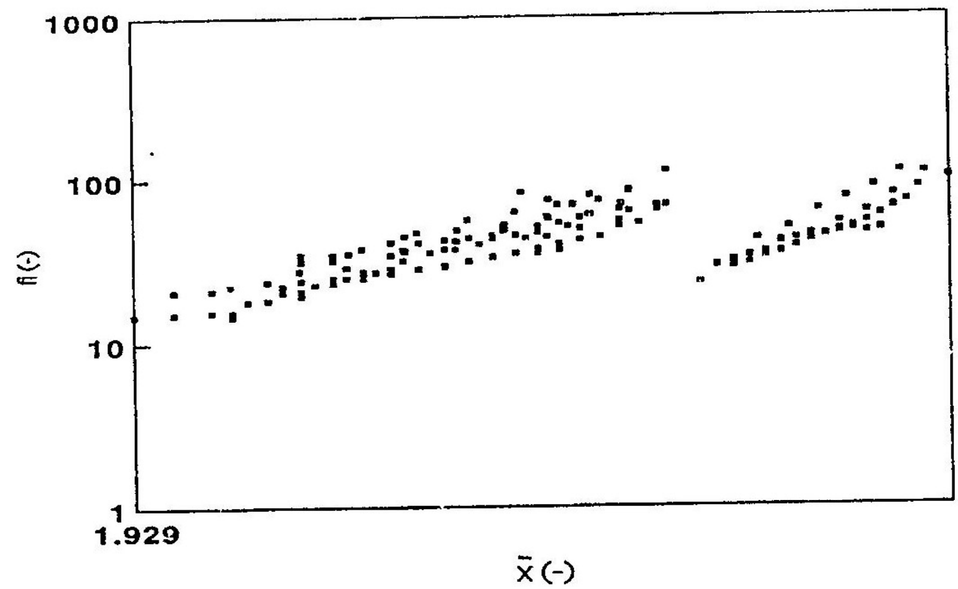

Fig. 8: Plot of fi versus flow path function - modified at obseved transition points only

CONCLUSIONS AND SIGNIFICANCE

A model has been derived and experimentally shown to be able to predict the minimum drawdown speed for floating particles which coupled the turbulence recirculation hypothesis. From experimental observations, the change in flow pattern due to change in macroscopic flow path does no occur for all cases with PBT as predicted. For this matter, the correlation in macroscopic flow path function should be applied only when an actual change is observed experimentally.

There is a noticeable similarity between floating solids drawdown and the suspension of settling solids for small density differences reported in this study (0 - 300 μm and medium particle sizes used (300 - 2500 kg/m^3). The position of an impeller has been modelled basing on recirculation pattern and has been able to account experimentally observed changes in a mechanically agitated system.

NOMENCLATURE

A	= Artea, m^2
A'	= partial area, m^2
C	= clearance
d	= distance, m
dp	= particle diameter,....
D	= impeller diameter, m
g	= gravitational due to gravity, ms^{-2}
Fb	= bouyancy force, N
FE	= eddy force, N
H	= height of liquid, m
N	= stirrer speed, s^{-1}
Njd	= just dispersed speed, s^{-1}
L	= macroscopic length scale, m
m	= mass, kg
P	= power consumption, W
P_o	= power number
T	= tank diameter, m
u	= r.m.s. relative velocity, ms^{-1}
V	= volume, m^3
V'	= partial volume, m^3
x	= macroscopic path function

$\bar{x}$ = modified macroscopic path
k_1, k_2 = constants
ρ = density, kg/m3
ρ_l = liquid density, kg/m3
ρ_s = solid density, kg/m3
Φ, Φ', f_i = pseudo constants
ε =energy dissipation per unit mass, W.kg-1
η = Kolmogoroff length scale, m
β = constant
v = insatantaneous fluctuating velocity in direction i, ms-1
$\Delta\rho$ = phase density difference, kgm-3
τ = shear stress, Nm-2

REFERENCES

1. T. N. Zwietering, Suspending of Solid Particles in Liquids by Agitators, *Chemical Engineering Science,* vol.8, 1958, pp 244-253.
2. A. W. Nienow, *Dispersion of Solids in Liquids , Mixing of Liquids by Mechanical Agitation.* J. Ulbrecht and G.Patterson (eds) 1985, pp 273-305.
3. J. Y. Oldshue, *Fluid Mixing Technology*, McGraw-Hill Co. New York, N.Y., 1983
4. J. H. Rushton, and J.Y. Oldshue, Mixing - Present Theory and Practice, *Chemical Engineering Progress*, vol.49, 1953, pp 161-168.
5. G. Baldi, R. Conti, E. Alaria, Complete Suspension of Particles in Mechanicaly Agitated Vessels, *Chemical Engineering Science*, vol. 33, 1978
6. Kolmogoroff, A. N., *C.R. Academic Science* U.S.S.R., vol. 30 1941, pp 301.
7. D.M. Levins and J.R. Glastonbury, Application of Kolmogoroff's Theory to Particle-Liquid Mass Transfer in Agitated Vessels, *Chemical Engineering Progress*, vol.27, 1972, pp 537-543.

8. P.N. Armenante, J.P. Mmbaga and R.R. Hemrajani, Mechanisms for the Entrainment of Floating Particles in Mechanically Agitated Liquids, *Proc. European Congress on Mixing*, Brugge-Belgium, Vol.II, 1991.
9. J.P. Mmbaga, Agitation requirements for complete drawdown of floating solids in stirred tanks, MSc. Thesis, New Jersey Inst. of Tech., 1989.
10. M. F. Edwards, D. I. Ellis, Draw-down of floating Solids into % Mechanically Agitated Vessels, *Mixing Symposium* Series No. 89 1987.
11. J. Susanto, Multiphase Solid-Liquid and Solid-Liquid-Gas Mixing, MSc. Thesis, New Jersey Inst. of Tech., 1989
12. H. Tennekes and J.L. Lumley, *A first course in Turbulence*, The MIT Press, Cambridge, MA, pp 262 1972

The manuscript was received 6th January 1994 and accepted for publication 6th December 1994