OPTIMUM GEOMETRICAL PARAMETERS FOR PRISMATIC FOLDED PLATE ROOFS: SYSTEMATIC SEARCH APPROACH

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ABSTRACT

Folded plate structures have unique structural advantages over conventional beam-and-slab structural forms. Selection of optimum geometrical parameters in design would make folded plate roofs more appealing to the national construction industry in terms of cost. This paper presents a study on shape optimization of prismatic folded plate roofs for minimum weight of the structure per unit covered area.

A computer based systematic search approach has been employed for the optimization study, while the transfer matrix method has been used for analysis.

Curves of optimum geometrical parameters versus span have been presented for three commonly used shapes of prismatic folded plate roofs. Economic comparison of the three shapes has been illustrated by plotting respective objective functions against spans.

INTRODUCTION

A prismatic folded plate structure is a shell consisting of a series of rectangular plates mutually supporting each other along common longitudinal edges and framing into transverse end diaphragms. Its structural merits are derived from spatialization and folding.

Folding a thin plate lengthwise takes the material of cross-section away from its middle plane and increases the lever arm of the bending stresses;

this increases the structural efficiency of folded plates over conventional beam-and-slab structural forms. Furthermore, by increasing the lever arm flexural rigidity of the structure is increased without significantly increasing the weight per unit area of the folded surface; therefore structural stability is enhanced by structure rather than by mass. For example (Fig. 1), a simply supported thin metallic plate will bend under its own weight; however the same plate, identically supported, but with folds following the direction of span will show higher stiffness and in consequence will bend much less. Increased structural efficiency and stability of folded plate structures make them ideal economical means of roofing large column-free areas for factory buildings, storage structures, convention halls, schools, etc.

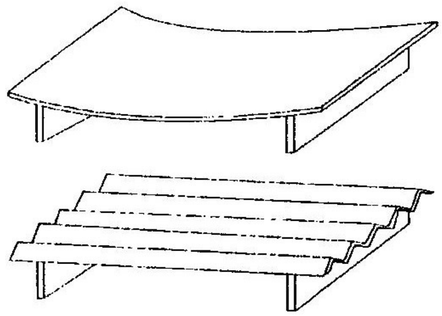

Fig. 1: Structural merits of a folded plate structure[1]

Considerable attention has been directed towards elastic analysis of prismatic folded plate roofs[1,2,3,4,5,6]. Both the so-called exact and approximate or simplified methods have been suggested for analysis of the structure. Exact methods are based on classical theories of mechanics and treat the structure as an elastic continuum; approximate methods of analysis incorporate simplifying assumptions in the classical analytical theories. Neither approximate nor exact methods indicate the degree of approximate function sufficient for an acceptable accurate solution, although exact methods do give more accurate solutions. In practice, however, folded plate structures can be safely designed using results from both the exact and approximate methods[6].

Few studies have been published on the optimization of folded plate structures. Several geometrical optimization approaches have been proposed, including trial and error methods, graphical methods and simplex numerical methods[7,8]. These methods are based on the understanding that an optimum solution to a linear programming problem, if it exists, can always be found in one of the basic feasible solutions.

Systematic search optimization approach has been employed in this study due to its programming simplicity, and the non-linearity aspect of the objective functions with at least five design variables. The objective function is an equation relating the weight of the structure to a set of geometrical parameters which satisfy constraint requirements; it is a scalar quantity which represents the most important single merit of design - weight. The systematic search method is based on the simple theory that the optimum solution to an optimization problem will be the one with the least value of the objective function.

This study has determined optimum geometrical parameters for minimum weight of structure per unit covered area of three commonly used shapes of prismatic folded plate roofs, the V-shape, the trough shape and the cylindrical shape, as shown in Fig. 2. It excludes the selection of type of material for minimum cost; only reinforced concrete folded plate roofs have been considered. A brief account of the quantity of steel as a cost function is considered.

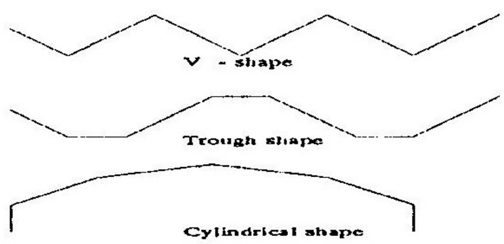

Fig. 2: Shapes of folded plate structures

ANALYSIS

The transfer matrix method of analysis as proposed by Holland[5] has been used in this study. The method makes full advantage of the modern digital computer. The general concept of this method is that a vector $[d,p]_{n,n}$ containing displacements and force elements at the edge n of the plate n (see Fig. 3) can be transformed into a corresponding vector $[d,p]_{n+1,n}$ of the same plate n at the edge $n+1$ by means of a transfer matrix $[F]_n$, which is also called a field matrix:

$$\begin{bmatrix} d \\ p \end{bmatrix}_{n+1,n} = [F]_n \begin{bmatrix} d \\ p \end{bmatrix}_{n,n} \tag{1}$$

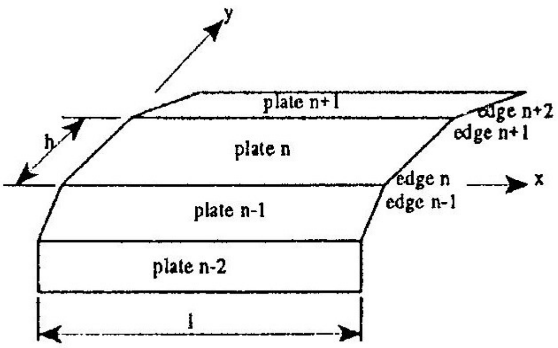

Fig. 3: Plate and edge definitions

Also the vector $[d,p]_{n+1,n}$ containing the displacement and force elements at edge $n+1$ in plate n can be transformed into the corresponding vector $[d,p]_{n+2,n+1}$ of the next adjoining edge $n+2$ of the next plate $n+1$ by means of a point matrix $[P]_i$:

$$\begin{bmatrix} d \\ p \end{bmatrix}_{n+2,n+1} = [P]_i \begin{bmatrix} d \\ p \end{bmatrix}_{n+1,n} \tag{2}$$

Thus the vector containing displacement and force elements at one end of a folded plate structure can be transformed step by step using the field and point matrices to the corresponding vector containing the displacement and force element at the other end of the folded plate. This vector has 9 elements; 4 of the elements relate to slab action of the structure (Fig. 4a), 4 elements represent plate action (Fig. 4b) and the remaining element is added in order to include the load term 1.

$$\begin{bmatrix} d \\ p \\ 1 \end{bmatrix} = \left[N_y N_{xy} N_x v w \theta M Q 1 \right]^T \tag{3}$$

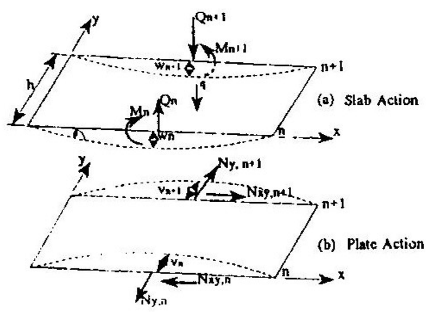

Fig. 4: Slab and plate forces

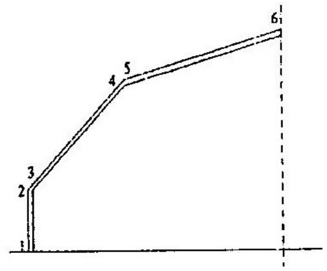

Fig. 5: Cross-section of a cylindrical folded plate roof

An example of applying the transfer matrix method is described below, by considering the cylindrical folded plate roof cross-section shown in Fig. 5.

The displacement and force vector at edge 1, $[d,p,1]_1$, can be transferred to edge 2 by a field matrix $[F]_1$; $[d,p,1]_2$ can also be transferred to edge 3 by a point matrix $[\mathbf{P}]_{23}$:

$$\begin{bmatrix} d \\ p \\ 1 \end{bmatrix}_2 = [F]_1 \begin{bmatrix} d \\ p \\ 1 \end{bmatrix}_1 \tag{4}$$

$$\begin{bmatrix} d \\ p \\ 1 \end{bmatrix}_3 = [P]_{23} \begin{bmatrix} d \\ p \\ 1 \end{bmatrix}_2 \tag{5}$$

By substituting (4) into (5) the following relationship is obtained:

$$\begin{bmatrix} d \\ p \\ 1 \end{bmatrix}_3 = [P]_{23}[F]_1 \begin{bmatrix} d \\ p \\ 1 \end{bmatrix}_1 \tag{6}$$

Hence $[P]_{23}[F]_1$ is the resultant matrix at edge 3. By applying the same concept the displacement and force element vector at edge 6 at the line of symmetry can be expressed in terms of the vector at edge 1 by alternative multiplication of field and point matrices:

$$\begin{bmatrix} d \\ p \\ 1 \end{bmatrix}_6 = [F]_3[P]_{45}[F]_2[P]_{23}[F]_1 \begin{bmatrix} d \\ p \\ 1 \end{bmatrix}_1 \tag{7}$$

Therefore if the vector $[d,p]_1$ at the first edge is known internal forces and displacements at any edge in the structure can be obtained by multiplying the vector with the resultant transfer matrix at the edge in question. The field and point matrices can be created from slab and plate action respectively.

Boundary Conditions

Due to boundary conditions at the free edges, N_y, N_{xy}, M and Q are known to be zero, and N_x, v, w and θ are the only unknowns. By applying boundary conditions at the line of symmetry of the structure the unknown elements of the vector at the free edge can be expressed in 4 equations, which can then be easily solved; these conditions are:

- Tangential stresses at the line of symmetry are zero, hence $N_{xy} = 0$;
- Horizontal displacement of the plate at the line of symmetry is zero, hence:

$$v.\sin\theta_{(5-6)} + w.\cos\theta_{(5-6)} = 0$$

- Rotation at edge 6, the line of symmetry, is zero, hence $\theta = 0$;
- The resultant vertical force at the line of symmetry, edge 6, is zero, hence:

$$Q_6 \sin\theta - N_6 \cos\theta - Applied.load_6 = 0$$

Computer Program

A computer program in PASCAL has been written for the analysis of prismatic folded plate structures using the transfer matrix method described above. It is a tailored program designed for purposes of geometrical optimization in this study, and for analysis of single span folded plate roof systems with simple support conditions. Loading is from reinforced concrete using grade 25 concrete, and a uniformly distributed imposed load of 0.75 kN/m^2; the analysis considers only the maximum loading case of the roof structure. Results from the program included longitudinal stresses and plate deflections, which were the necessary requirements for this study. The flow chart for the program is illustrated in Fig. 6.

OPTIMIZATION

Pre-optimization requirements include the establishment of three important attributes: design variables in the form of geometrical parameters, constraints to be applied to the geometrical parameters, and the objective function. These have been established for each of the three shapes chosen for optimization.

Constraints for geometrical parameters have been prescribed in consideration of practical structural and construction aspects[9], such as minimum thickness to accommodate reinforcement, reinforcement cover, serviceability and the placing of concrete. It is also ensured that the maximum

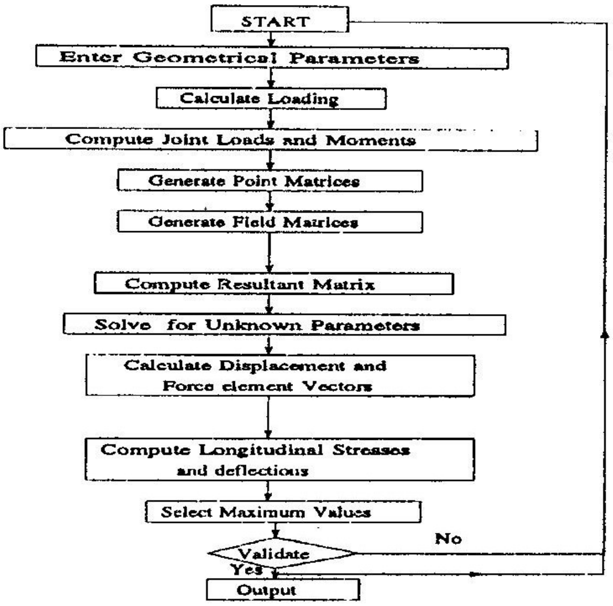

Fig. 6: Flow Chart for analysis of Prismatic folded plate roofs using the transfer matrix method

absolute value of compressive stress developed in the folded plate roof does not exceed permissible design compressive stresses in concrete. It has been assumed that all compressive stresses are taken care of by the concrete alone; there is no steel inclusion in the compression zone to enhance its strength capacity.

Influence of quantity of reinforcement steel

Reinforcement steel can contribute significantly to the cost of a reinforced concrete structure. Ideally, for structural optimization, the quantity of steel should have been an independent variable. However in order to limit the number of variables steel has been made to be a dependent variable, the immediate variation of which does not influence forces in the structure. In order to control the area of steel in the optimized folded plate structure a simple study was carried out to determine the cptimum percentage of steel which would not cause a significant rise in the total cost of the cross section relative to the cost component contributed by concrete. It was found that the optimum percentage of steel below which the quantity of steel does not cause a significant rise in cost is 0.75%. Since the percentage of steel is limited to 0.75% the allowable tensile stress in the folded plate structure should not exceed 15 N/mm^2.

Systematic Search

The geometrical parameters which would give the optimum solution to an objective function were searched systematically using a computer program prepared especially for this study. One of the main functions of the program is to validate geometrical parameters before computing the objective function. The geometrical parameters are validated on the basis that levels of stress and deflection in the structure, based on a given set of geometrical parameters, do not exceed set limits. Another function of the program is to carry out different combinations of geometrical parameters within limits set by geometrical constraints, before validation. For a particular span the program adopts geometrical parameters with the least objective function as optimum. The flow chart for the search program is illustrated in Fig. 7.

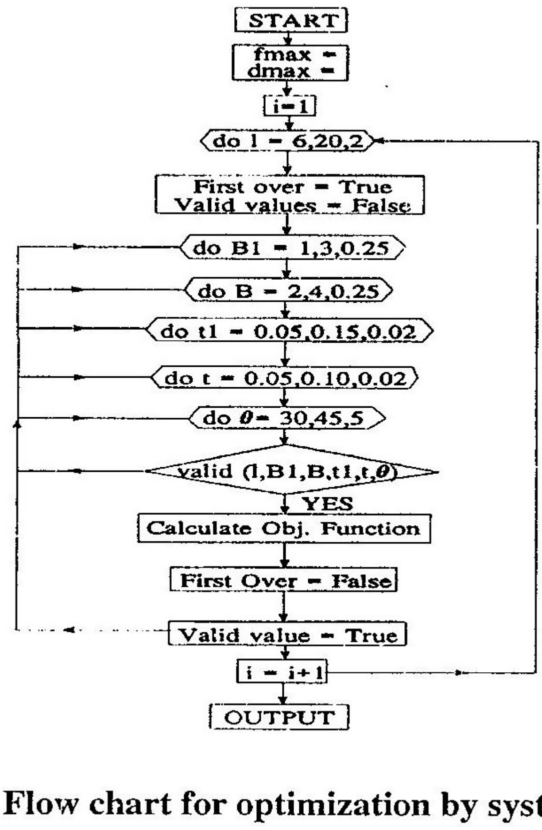

Fig. 7: Flow chart for optimization by systematic search

V-shaped folded plate roof

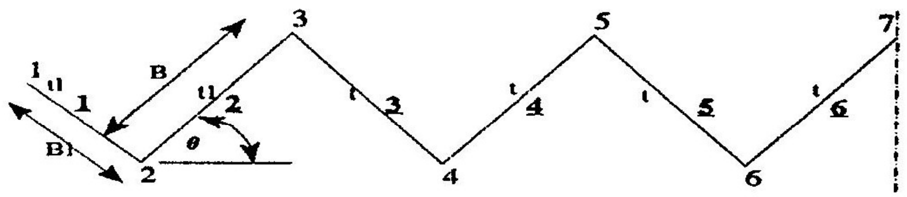

Fig. 8: Design variables for the V-shaped roof

Table 1: Design variables and constraints for V-shaped roof (Fig. 8)

Design Variables		Constraints	
Variable	Description	min. value	max. value
B1	width of outermost plates [m]	1.0	3.0
B	width of inner plates [m]	2.0	4.0
t1	thickness of outermost plates [m]	0.05	0.15
t	thickness of inner plates [m]	0.05	0.10
θ	angle of plate inclination [°]	30	45

Behaviour constraints:

Maximum tensile stress = 7 N/mm^2

Maximum allowable deflection = span/500

The Objective function for this shape is derived as follows:

$$Objective.function = \frac{Total.Weight}{Covered.area}$$

$$= \frac{(t_1 \times B_1 \times l \times \rho) + (t_1 \times B \times l \times \rho) + (4 \times t \times B \times l \times \rho)}{l \times (B_1 + 5B_1) \times \cos\theta}$$

Trough-shaped folded plate roof

the

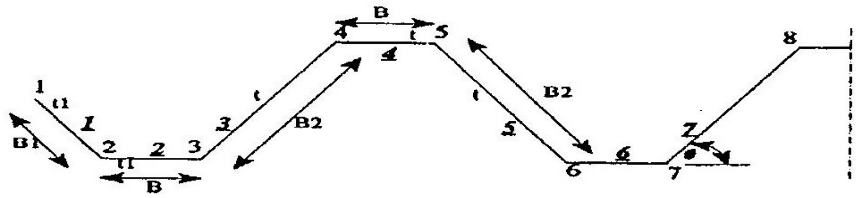

Fig. 9: Design variables for the trough-shaped roof

Table 2: Design variables and constraints for Trough roof (Fig. 9)

Design Variables		Constraints	
Variable	Description	min. value	max. value
B1	width of outermost plates [m]	1.0	3.0
B	width of inner horizontal plates [m]	0.25	2.0
B2	width of inner inclined plates [m]	2.0	4.0
t1	thickness of outermost plates [m]	0.05	0.15
t	thickness of inner plates [m]	0.05	0.10
θ	angle of plate inclination [°]	30	45

Behaviour constraints:

Maximum tensile stress = 6 N/mm^2

Maximum allowable deflection = span/500

The Objective function for this shape is derived as follows:

$$Obj.\ Func. = \frac{(t_1 \times B_1 \times l \times \rho) + (t_1 \times B \times l \times \rho) + (\frac{5}{2} \times t \times B \times l \times \rho) + (3 \times t \times B_2 \times l \times \rho)}{l \times (B_1 + 3B_2) \times \cos\theta + \frac{7}{2} B \times l}$$

Cylindrical folded plate roof

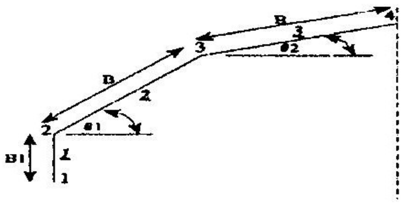

Fig. 10: Design variables for a cylindrical roof

Table 3: Design variables and constraints for Cylindrical roof (Fig. 10)

Design Variables		Constraints	
Variable	Description	min. value	max. value
B1	width of outermost plates [m]	1.0	3.0
B	width of inner horizontal plates [m]	1.0	2.0
t1	thickness of outermost plates [m]	0.05	0.15
t	thickness of inner plates [m]	0.05	0.10
$\theta 1$	angle of plate inclination [°]	25	45
$\theta 2$	angle of plate inclination [°]	0	25

Behaviour constraints:

Maximum tensile stress = 6 N/mm^2
Maximum allowable deflection = span/500

The Objective function for this shape is derived as follows:

$$Obj.Func. = \frac{(t_1 \times B_1 \times l \times \rho) + (2 \times t \times B \times l \times \rho)}{l \times (B \times \cos\theta_1 + B \times \cos\theta_2)}$$

RESULTS

Results of optimization are shown in Tables 1, 2 and 3. The relationships between objective function and span are given in Fig. 11, for the three shapes of prismatic folded plate roofs considered in this study. Curves of geometrical parameters versus spans are illustrated in Figures 12, 13 and 14.

Table 4: Optimum geometrical parameters for V-shaped roof

Span [m]	B1 [m]	B [m]	t1 [m]	t [m]	θ [deg]	Objective Function [kN/m²]	Maximum deflection [m]	Max tensile stress [N/mm²]
6.0	1.00	2.00	0.050	0.050	30.00	1.386	0.00184	3.0510
8.0	1.00	2.00	0.050	0.050	30.00	1.386	0.00185	5.4136
10.0	1.00	3.00	0.050	0.050	30.00	1.386	1.00930	6.5364
12.0	2.50	3.00	0.050	0.050	30.00	1.386	0.00929	6.8644
14.0	3.00	3.50	0.050	0.050	35.00	1.465	0.01717	6.9154
16.0	3.00	4.00	0.075	0.050	35.00	1.688	0.02934	6.7309
18.0	3.00	4.00	0.075	0.050	45.00	1.955	0.02936	6.9095
20.0	3.00	4.00	0.125	0.050	45.00	3.062	0.01048	6.7640

Table 5: Optimum geometrical parameters for trough-shaped roof

Span [m]	B1 [m]	B2 [m]	B [m]	t1 [m]	t [m]	θ [deg]	Objective Function [kN/m²]	Maximum deflection [m]	Max tensile stress [N/mm²]
6.0	3.00	4.00	0.250	0.050	0.050	30.00	1.462	0.00213	4.7576
8.0	3.00	4.00	0.250	0.050	0.050	30.00	1.462	0.00089	4.1598
10.0	3.50	4.00	0.250	0.050	0.050	30.00	1.462	0.00097	4.5951
12.0	3.00	4.00	0.250	0.050	0.050	30.00	1.462	0.00112	5.6209
14.0	3.00	4.00	0.250	0.050	0.050	40.00	1.652	0.00108	5.8209
16.0	2.50	4.00	0.250	0.075	0.050	45.00	1.952	0.00088	5.9681
18.0	1.50	4.00	1.250	0.100	0.050	40.00	2.344	0.00169	5.7994
20.0	2.00	4.00	1.250	0.125	0.075	40.00	3.345	0.00162	5.6256

Table 6: Optimum geometrical parameters for cylindrical roof

Span [m]	B1 [m]	B [m]	t1 [m]	t [m]	θ [deg]	θ [deg]	Objective Function [kN/m²]	Maximum deflection [m]	Maximum tensile stress [N/mm²]
6.0	1.00	1.75	0.050	0.050	25.00	5.00	1.622	0.00008	0.9759
8.0	1.00	1.75	0.050	0.050	30.00	5.00	1.657	0.00024	1.3938
10.0	1.50	1.75	0.050	0.050	35.00	15.00	1.921	0.00026	1.3993
12.0	2.00	1.75	0.050	0.050	35.00	20.00	2.144	0.00050	1.4271
14.0	2.50	1.75	0.050	0.050	45.00	20.00	2.498	0.00028	1.4389
16.0	2.50	1.75	0.075	0.050	40.00	15.00	2.870	0.00032	1.4709
18.0	3.00	1.75	0.075	0.050	35.00	20.00	3.119	0.00071	1.4675
20.0	3.00	1.75	0.125	0.050	35.00	10.00	4.181	0.00103	1.4813

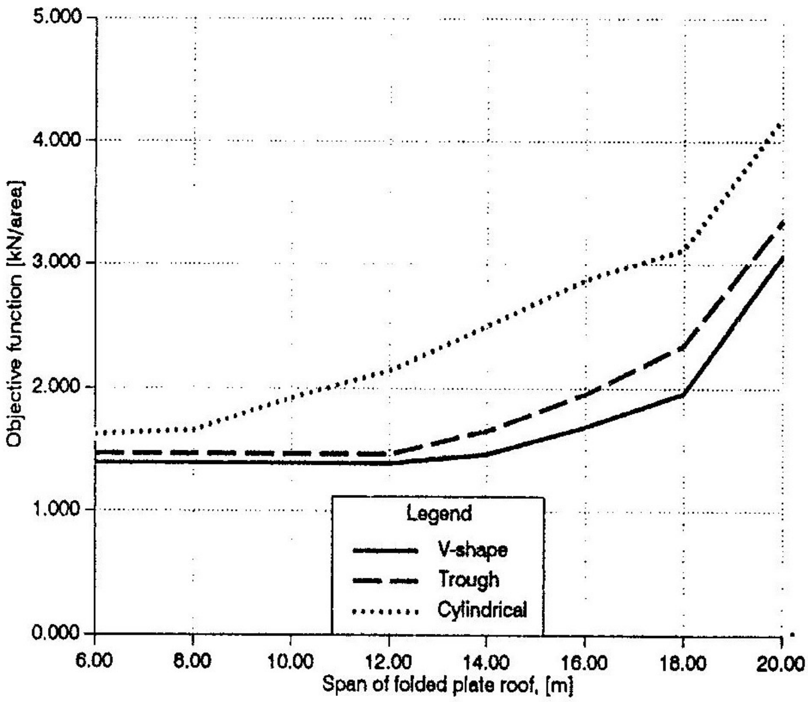

Fig. 11: Relationship between Objective Function and Span

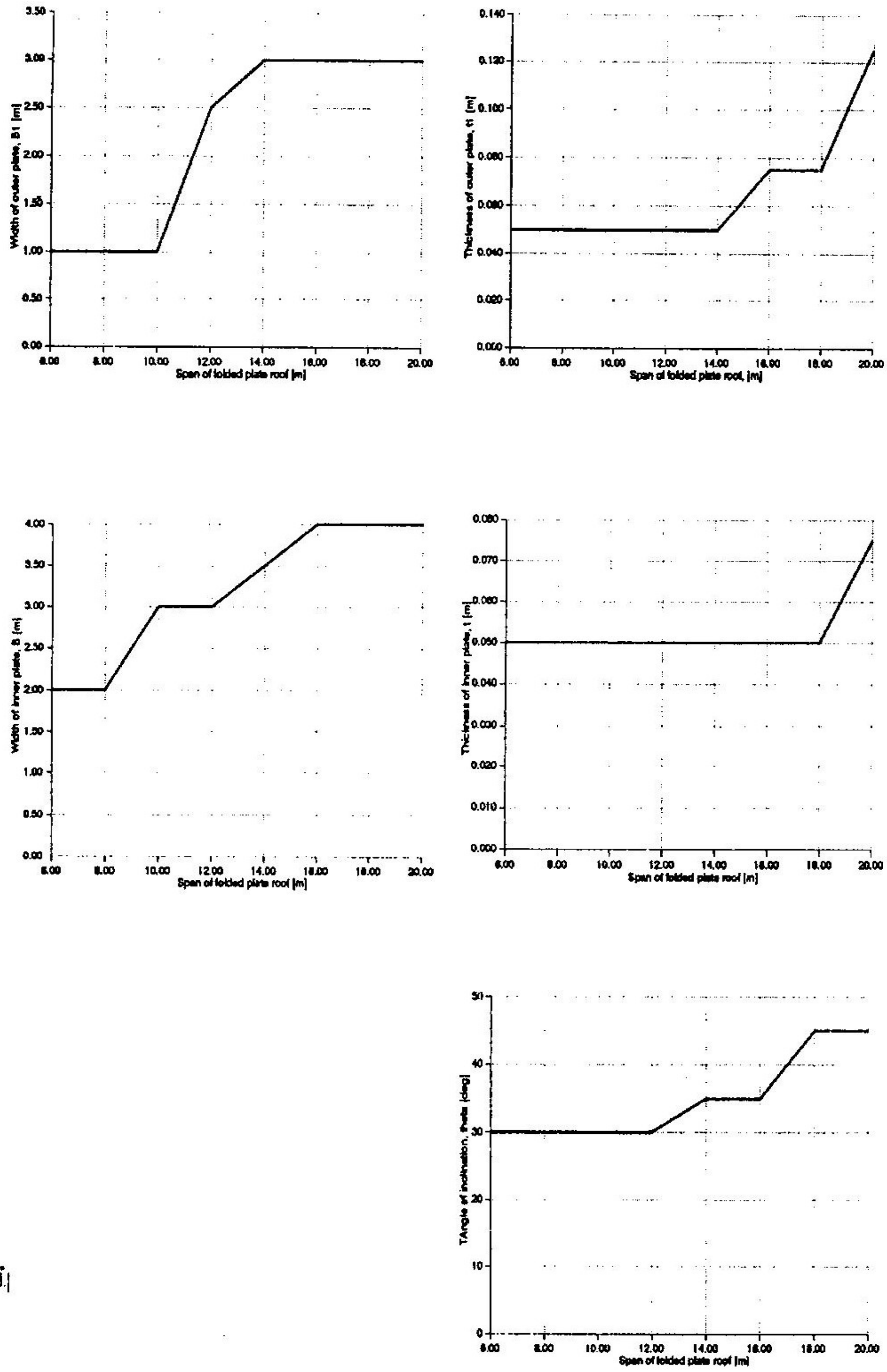

Fig. 12: Optimum geometrical parameters for V-shaped folded plate roof

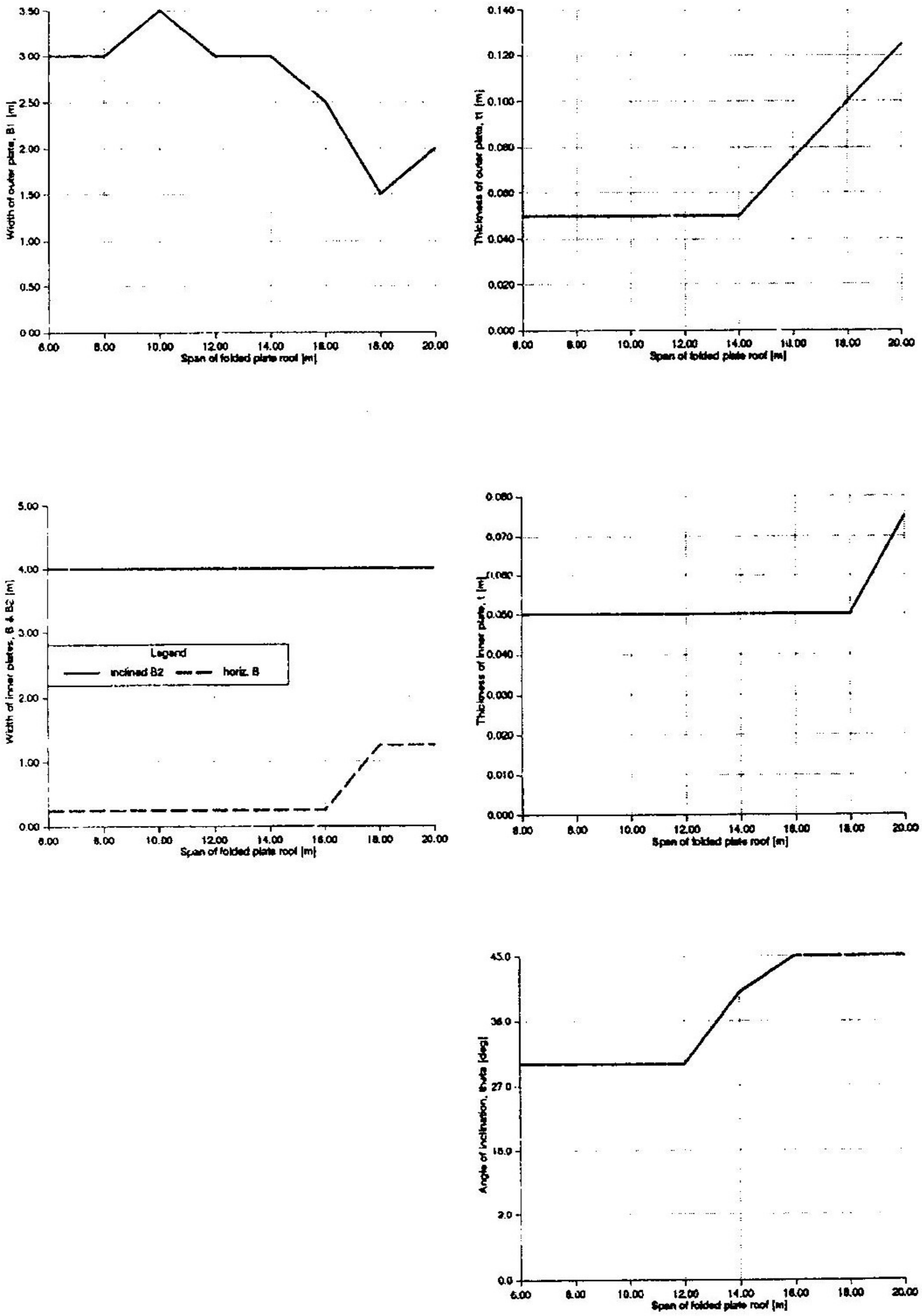

Fig. 13: Optimum geometrical parameters for Trough-shaped folded plate roof

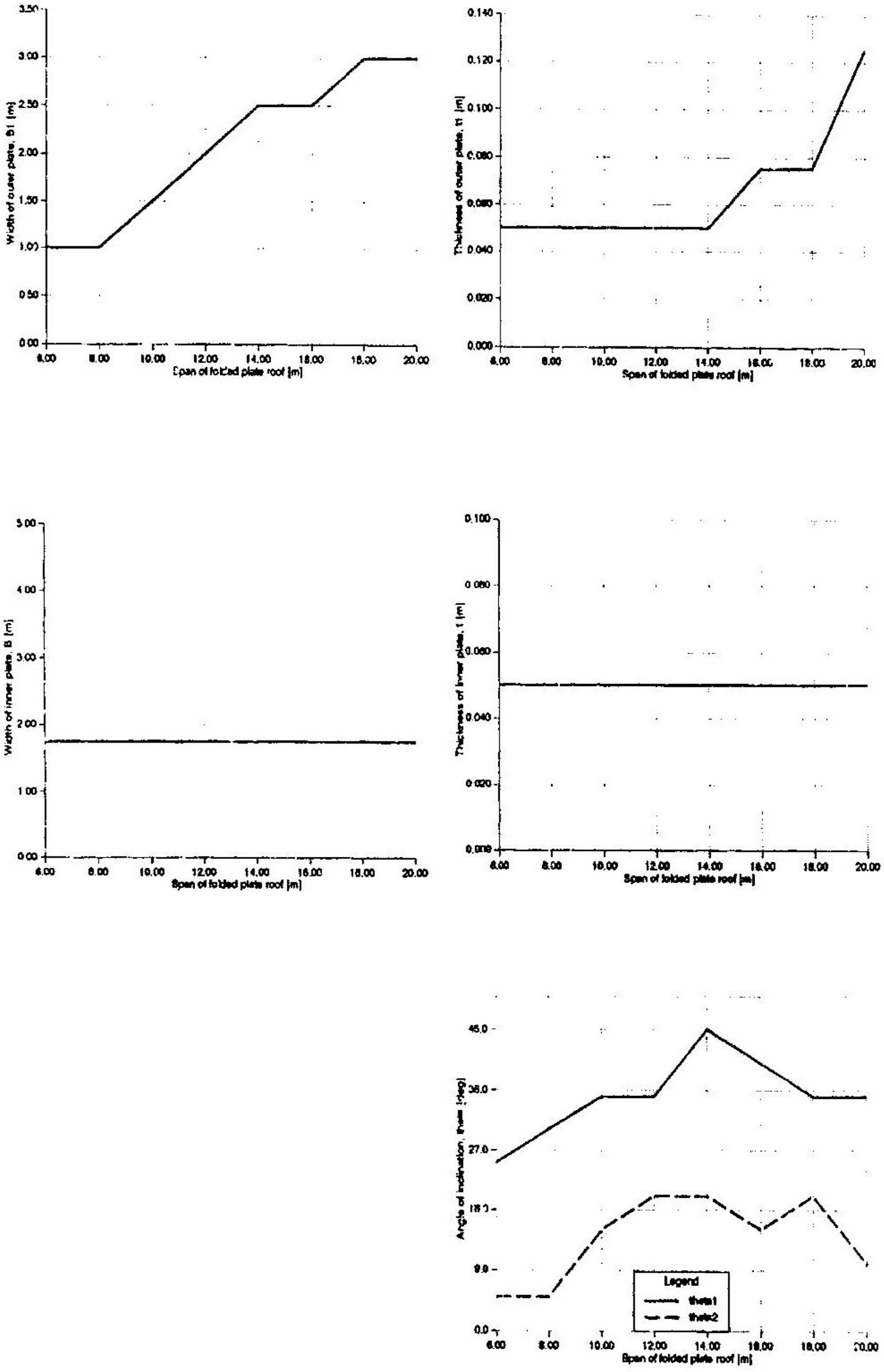

Fig. 14: Optimum geometrical parameters for cylindrical folded plate roof

CONCLUSION

Results show that there is a general increase in the geometrical parameters as the span increases, especially for V-and trough-shaped folded plate roofs. This is expected, since with increasing span stresses and deflections in the structures increase. Furthermore the objective functions of both the V- and trough-shaped folded plate roofs drastically increase from a span of about 16 metres. This implies that for spans exceeding 16 metres these shapes may not be economical. Thus in order to take full advantage of minimum cost per unit covered area, spans should be limited to 16 metres by introducing intermediate diaphragms.

The V-shaped prismatic folded plate roof has the lowest objective function, hence it is more economical compared to the other two shapes.

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