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MATHEMATICAL MODELLING OF ROAD BUMPS

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Abstract

In this paper, a general mathematical model that may be used for construction of a road bump on an inclined road is developed. It is shown that if the parameters for the inclination are set to zero, the resulting model reflects the special case of a flat road. Existing experiences are briefly highlighted and a case study was done on different locations along the University Road to demonstrate the developed model.

Introduction

A common way of discouraging speeding on housing estates, campuses and other restricted road systems is the construction of road bumps or humps. Road bumps or humps are roadway geometrical design features intended to physically reduce vehicle operating speed in order to improve traffic and residents safety. It is a method of traffic management to achieve speed restrictions by influencing each driver's behaviour crossing them. Road bumps or humps consist of raised pavement constructed or placed in, on and across or partly across a roadway to reduce the speed of vehicles travelling along that roadway. Non motorized transport estimate of the vehicle speed becomes less reliable when the vehicle speed is high. To avoid this risk of estimating speed, speed calm measures are important in urban areas.

A road bump is differentiated from a road hump by the fact that a road hump normally has a maximum height of 7.6cm to 10cm and a travel length of 3.7m resulting in most vehicles slowing down to about 24km/h whereas a road bump is generally 7.5 cm. to 15.2 cm in height with a length of 0.3m to 1m resulting in vehicles slowing down to near 8km/h or less [1].

Huntley & James [2] presented a mathematical model that could be used for construction of a road bump on a flat road. In their model, they aimed at estimating the shock on the body of the car transmitted by the suspension system when the car was driven over a curved bump. No distinction was made between the wheels of the car so the effect of the bump on an isolated single wheel was studied. The following assumptions were considered:

- Speed of the car, v, is constant;
- Shape of the curve could be described by the cosine curve $z = a\left(1 - \cos\frac{\pi x}{L}\right)$ (see figure 1 below);
- The suspension of a vehicle can be modelled by a spring with constant k and natural length l;
- Motion over a bump is governed by Newton's Laws;
- No damping involved.

Suggested model

The general layout is given in figure 1 below [2].

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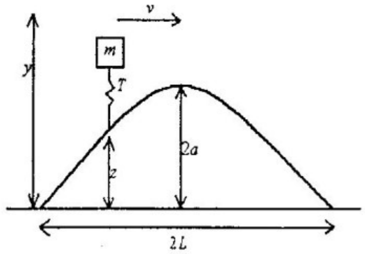


Fig.1:General arrangement of a car passing over a bump on a flat road.

Equations of motion are thus

$$\begin{cases} m\ddot{y} = -mg - T \\ T = k(y - z - l) \end{cases} \quad (1)$$

or $m\ddot{y} + ky = -mg + kz + kl$ (2)

In equilibrium position we have

$$kY = -mg + kl \quad (3)$$

With a change of variable $u = y - Y$ in Eq. (2) we obtain $m\ddot{u} + ku = kz$

or

$$m\ddot{u} + ku = ka\left(1 - \cos\frac{\pi x}{L}\right) = ka\left(1 - \cos\frac{\pi vt}{L}\right) \quad (4)$$

With the initial conditions $u = 0, \dot{u} = 0$, Eq.(4) has a solution

$$u = a(1 - \cos\alpha t) + ka\frac{\left(\cos\alpha t - \cos\frac{\pi vt}{L}\right)}{\left(k - \frac{m\pi^2 v^2}{L^2}\right)} \quad (5)$$

where $\alpha = \sqrt{\frac{k}{m}}$.

Using Eqs (4) and (5), it can be shown that

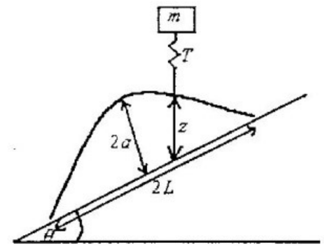
$$m\ddot{u} = -k\left[a(1 - \cos\alpha t) + ka\frac{\left(\cos\alpha t - \cos\frac{\pi vt}{L}\right)}{\left(k - \frac{m\pi^2 v^2}{L^2}\right)}\right] + ka\left(1 - \cos\frac{\pi vt}{L}\right) \quad (6)$$

Eq. (6) is the force experienced by the wheel over the bump constructed on a flat road. In this paper it is intended to develop a more general mathematical model which may be used for construction of a road bump on an inclined road.

The model of a road bump on an inclined road

The general layout is given in figure 2 below.

Fig. 2:Generalarrangement of a car



passing over a bump on an inclined road.

The assumptions stated in the introduction section are still valid with an exception that the shape of the curve is now described by a tilted cosine function

$$z = a\left[1 - \left(\cos\frac{\pi x}{L} + \theta\right)\right] \quad (7)$$

With the tilted cosine function, equation (4) modifies to

$$m\ddot{u} + ku = ka\left(1 - \theta - \cos\frac{\pi vt}{L}\right) \quad (8)$$

With the initial conditions $u = 0, \dot{u} = 0$, equation (8) has a solution

$$u = (a - a\theta)(1 - \cos\alpha t) + ka\frac{\left(\cos\alpha t - \cos\frac{\pi vt}{L}\right)}{\left(k - \frac{m\pi^2 v^2}{L^2}\right)} \quad (9)$$

Using Eqs. (8) and (9) it can be shown that the force experienced by the wheel over the bump constructed on an inclined road is

$$m\ddot{u} = -k\left[a(1 - \theta)(1 - \cos\alpha t) + ka\frac{\left(\cos\alpha t - \cos\frac{\pi vt}{L}\right)}{\left(k - \frac{m\pi^2 v^2}{L^2}\right)}\right] + ka(1 - \theta) - ka\cos\frac{\pi vt}{L} \quad (10)$$

Note that if we set $\theta = 0$, Eq. (10) reduces to Eq. (6), this being the case of a flat road consideration.

Estimation of parameters

Bumps are devices that force the drivers to travel at extremely low speed ($<10 km/hr$) by being very short and high. The basic design parameters of bumps are length of the bump, height of the bump and spacing between successive bumps. Some of the recommended dimensions for the road bumps in some countries are summarized below[1],[3-5]:

Table 1: Existing dimensions of road bumps in UK and USA.

Parameter	UK	USA
Height (mm)	$35-60$	$76-152$
Length (m)	$0.9-1.0$	$0.3-0.9$
Speed reduction ($<10 km/hr$)	≤ 10	≤ 8

To minimize accidents on a housing estate, it is wise to use appropriately spaced speed bumps to restrict all vehicles to some maximum speed, say v_1. If we assume that a single bump can reduce a vehicle's speed to v_0 for the next bump at a separation distance D, then our aim is to choose v_0 and D to prescribe v_1. The vehicle goes over the first speed bump at a speed v_0, accelerates up to a speed v_1 and slows down to v_0 for the next bump at a separation distance D.

The combination of v_0 and D will vary from vehicle to vehicle. Huntley & James [1] suggested that if we decide to design for the worst case, we may consider a sports car that is able to accelerate fast and decelerate fast. Let the acceleration be a and deceleration be $-b$ (both assumed to be constant). The

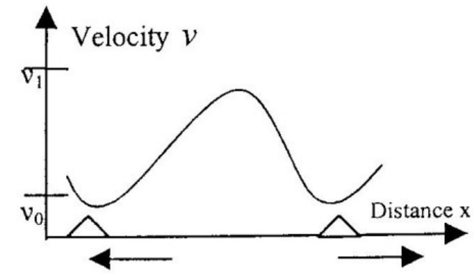


Figure 3: Velocity behaviour for a vehicle crossing two successive speed bumps.

formula for constant acceleration gives

$$D = \frac{v_1^2 - v_0^2}{2}\left(\frac{1}{a} + \frac{1}{b}\right) \qquad (11)$$

Taking a $0-100 km/hr$ of $10 \sec$ and assuming constant acceleration throughout we have $a = 10 km/hr^2$. The Highway Code estimates a constant deceleration of $b = 23.4 km/hr^2$ when evaluating its braking distance. Thus

$$D = 0.24\left(v_1^2 - v_0^2\right)m. \qquad (12)$$

If instead, we are given D, we can use the formula to find v_0. A reasonable value would be $D = 45m$. Thus

$$v_1^2 = v_0^2 + 0.19. \qquad (13)$$

For the minimum value of $8 km/hr$, this gives $v_1 = 57 km/hr$.

A case study

A case study was carried on different locations along the University Road specifically at places where the model will suit i.e. the sloping parts of the road. The areas of concentration were:

- The sloping part at Rwegarulila Water Resources Institute, commonly known as "Gate Maji";
- The sloping part near the Vice-Chancellor's Residence;
- The sloping part between the Administration Block and Hall of Residence No. 7;
- UDASA area;
- The sloping part between UCLAS and the Makongo Drive.

The table below summarizes the results observed on a Daladala bus:

Table 2: Summary of findings along University Road

Area	Approach speed Near the bump $(km/hr$	Cross over speed (km/hr)	Maximun speed between two bumps $(km/hr$
VC's Residence	10-15	5-6	40-45
"Gate Maji"	10-15	5-6	40-45
Admin. Block/Hall 7	15-20	6-8	40-45
UDASA	20-25	7-10	40-45
UCLAS/ Makongo Drive	20-25	7-10	40-45

The cross over speeds at VC's residence and "Gate Maji" is lower than the observed speeds at the other bumps because of the corner between the two bumps. Assume a typical mass of a car crossing on these bumps to be $1000kg$ so that $m = 250kg$. The observed average crossing over speed is $v = 5.5km/hr$. Also we assume the heights and lengths of the bumps to be $2a = 0.2m$ and $2L = 0.4m$ respectively. An average value of k can be taken as $k \approx 900$. If we assume that the approximate inclination of the piece of road between the VC's residence and "Gate Maji" is $20^\circ = \pi/9$ *radians*, then by Eq. (10), the force experienced by the wheel over the bump is

$$m\ddot{u} = 58.5\cos(1.9t) - 90\cos(23t) \quad (14)$$

The force in Eq. (14) will cause discomfort to the driver and thus discourage him from driving fast. For a very short time, say, $t = 0.2\sec$, $m\ddot{u} \approx 286kgf$, and for $t = 0.4\sec$, $m\ddot{u} \approx 129.42kgf$.

Conclusions

The purpose of this paper was to formulate a mathematical model that may be used for construction of road bumps on an inclined road. We have also shown that if the inclination parameters are set to zero, the model exactly reflects the case of a flat road. The case study carried along the University road indicates that road bumps are necessary on a residential sloping road since the vehicles, especially the business ones, are driven on high speeds.

References

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