ANALYSIS ON A POSSIBLE SOLUTION TO DECREASING THE EFFECT OF NON-UNIFORM AIR-GAPS DUE TO ROTOR ECCENTRICITY IN ASYNCHRONOUS MACHINES

Dominic J. Chambega
Department of Electrical Engineering, P.O. Box 35131,
Dar es- Salaam, TANZANIA

Rotor eccentricity introduces non-uniformity of the airgap which in turn causes additional frictional torques and losses in addition to establishing a uni-directional pulling magnetic force which is disastrous to the machine. This paper analyses the extent of the damage to the machine due to non-uniformity of the air-gap and proposes increasing the number of poles as a possible solution to arrest the effect. It is found through the analysis that as the number of poles increases the effect of eccentricity decreases.

Keywords: Eccentricity, air-gap, magnetic induction, harmonics.

INTRODUCTION

If the airgap is uniform a sinusoidal magnetic induction curve is obtained along the airgap. This in turn produces forces that are uniformly distributed with no extra stress to the machine. Due to human errors, often during the construction and assembly stage, the airgap is not uniform causing a number of undesirable features including non-uniformity of magnetic induction which in turn may result in isolated magnetic saturated spots within the machine in addition to producing a unidirectional magnetic pulling force. This paper analyses how this later effect arises and determines the magnitude of the damage to the machine and proposes a solution to arrest the effect.

ANALYSIS OF THE EFFECT OF ECCENTRICITY

In order to access the effect of eccentricity of the rotor Figure 1 is considered. For any point on the circle circumference the following expression is true:

$$\rho = R - \delta; x = \rho\cos\varphi; y = \rho\sin\varphi \qquad (1)$$
$$\textit{from where} \quad \rho = \varepsilon\cos\varphi + \sqrt{\varepsilon^2\cos^2\varphi + r^2 - \varepsilon^2}$$

For an ideal machine the following equation is valid (Figure 1 (a))

$R = r + \delta_0$ where δ_0 - nominal airgap length

Introducing the per unit (p.u.) system results into:

$$\delta_{pu} = 1 + r_{pu} - \varepsilon_{pu}\cos\varphi - \sqrt{\varepsilon_{pu}^2\cos^2\varphi + r_{pu}^2 - \varepsilon_{pu}^2} \qquad (2)$$
$$\textit{where}\ \delta_{pu} = \frac{\delta}{\delta_o};\ \varepsilon_{pu} = \frac{\varepsilon}{\delta_o};\ r_{pu} = \frac{r}{\delta_o}$$

In practice, a real electrical machine has $r_{pu} >> 1$, $\varepsilon_{pu} \le 1$, then

$$\sqrt{\varepsilon_{pu}^2\cos^2\varphi + r_{pu}^2 - \varepsilon_{pu}^2} \approx r_{pu} \qquad (3)$$

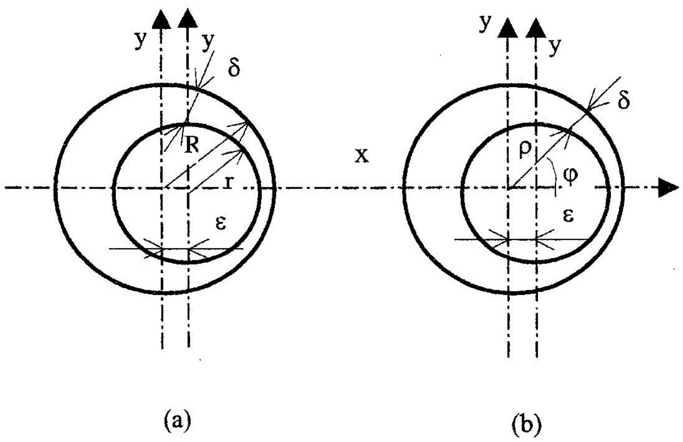

Figure 1: Position of the rotor inside the stator bore showing eccentricity.

The error obtained experimentally showed that its maximum value does not exceed 1% [1].

The power line in the airgap has to be normal to the stator bore and to the outer circumference of the rotor. In Figure 1 the line representing the line of action of magnetic force is not normal to the circumference of the rotor. But because $\varepsilon \cos \varphi << 1$ the error here is very small. Hence

$$\delta_{pu} = 1 - \varepsilon_{pu} \cos \varphi \tag{4}$$

Magnetic conductance of the airgap is inversely proportional to its length.

$$\lambda_{pu} = \frac{1}{\delta_{pu}} = \frac{1}{1 - \varepsilon_{pu} \cos \varphi} \tag{5}$$

For cases when the machine is not saturated magnetic induction in the airgap equals [2]:

$$B = \frac{B_o \sin(\omega t - \varphi_1}{1 - \varepsilon_{pu} \cos \varphi} \tag{6}$$

Where B_o - the amplitude value of magnetic induction at nominal length of airgap.

$$\varphi_1 = p\varphi ;$$

$$\frac{B(p, \varphi, \omega t)}{B_o} = \frac{\sin(\omega t - p\varphi)}{1 - \varepsilon_{pu} \cos \varphi} \tag{7}$$

Introducing

$$B = \frac{(p, \varphi, \omega t)}{B_o} = B_{pu}(p, \varphi, \omega t) \tag{8}$$

Then

$$B_{pu}(p, \varphi, \omega t) = \frac{\sin(\omega t - p\varphi)}{1 - \varepsilon_{pu} \cos \varphi} \tag{9}$$

Expression (9) can be rewritten as follows:

$$B_{pu}(p\varphi, \omega t) = F_1(p\varphi)\sin\omega t - F_2(p\varphi)\cos\omega t$$
$$\text{where } F_1(p\varphi) = \frac{\cos p\varphi}{1-\varepsilon_{pu}\cos\varphi}; \quad \text{and } F_2(p\varphi) = \frac{\sin p\varphi}{1-\varepsilon_{pu}\cos\varphi} \tag{10}$$

In practice $\varepsilon_{pu} < 1$ hence the function $F_1(p, \varphi)$ and $F_2(p, \varphi)$ are continuous and therefore it is possible to expand them into Fourier series [3]. The result of this expansion is an expression for magnetic induction in the airgap in the form of a sum of harmonic components:

$$F_1(p, \varphi) = \frac{a_o}{2} + \sum_{n=1}^{\infty}(a_n \cos n\varphi + b_n \sin n\varphi)$$
$$F_2(p, \varphi) = \frac{c_o}{2} + \sum_{n=1}^{\infty}(c_n \cos n\varphi + d_n \sin n\varphi) \tag{11}$$

RESULTS

Consider only the first six terms of the series because the amplitudes of the terms reduce as the number of harmonics increase and that the series converges and the effect of higher harmonics with small amplitudes on the performance of the machine is negligible. The zero terms are given by

$$a_o = \frac{1}{\pi}\int_o^{2\pi} \frac{\cos p\varphi d\varphi}{1 - \varepsilon_{pu}\cos\varphi}; \qquad c_o = \frac{1}{\pi}\int_o^{2\pi} \frac{\sin p\varphi d\varphi}{1 - \varepsilon_{pu}\cos\varphi}$$

The solution of these integrals show that B_{0pu} does not depend on φ. This means that along the whole stator bore flux has the same direction and therefore it will be very small since it has to find an extra route for closing (shaft, side covers, etc). Therefore the zero terms of the expression can be neglected.

The rest of the terms in the series are determined using the following expressions:

$$a_n = \frac{1}{\pi}\int_o^{2\pi} \frac{\cos p\varphi \cos n\varphi}{1 - \varepsilon_{pu}\cos\varphi} d\varphi; \qquad b_n = \frac{1}{\pi}\int_o^{2\pi} \frac{\cos p\varphi \sin n\varphi}{1 - \varepsilon_{pu}\cos\varphi} d\varphi$$

$$c_n = \frac{1}{\pi}\int_o^{2\pi} \frac{\sin p\varphi \cos n\varphi}{1 - \varepsilon_{pu}\cos\varphi} d\varphi; \qquad d_n = \frac{1}{\pi}\int_o^{2\pi} \frac{\sin p\varphi \sin n\varphi}{1 - \varepsilon_{pu}\cos\varphi} d\varphi$$

Having solved these integrals the following expressions for magnetic induction in the airgap are obtained:

$$\begin{aligned} B(p,\varphi,\omega t) = {} & B_1' \sin(\omega t - \varphi) + B_2' \sin(\omega t - 2\varphi) + \\ & + B_3' \sin(\omega t - 3\varphi) + B_4' \sin(\omega t - 4\varphi) + \\ & + B_5' \sin(\omega t - 5\varphi) + B_1'' \sin(\omega t + \varphi) + \\ & + B_2'' \sin(\omega t + 2\varphi) + B_3'' \sin(\omega t + 3\varphi) + \\ & + B_4'' \sin(\omega t + 4\varphi) + B_5'' \sin(\omega t + 5\varphi) \end{aligned} \tag{12}$$

For a two-pole machine the amplitude of the magnetic induction is determined by the following expressions:

$$B' = B_0 \frac{1}{\sqrt{1-\varepsilon^2_{pu}}}; \qquad B_2' = B_0 \frac{1-\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon_{pu}\sqrt{1-\varepsilon^2_{pu}}};$$

$$B_3' = B_0 \frac{2-\varepsilon^2_{pu}-2\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon^2_{pu}\sqrt{1-\varepsilon^2_{pu}}}; \qquad B_4' = B_0 \frac{4-3\varepsilon^2_{pu}-(\varepsilon^2_{pu}-4)\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon^2_{pu}\sqrt{1-\varepsilon^2_{pu}}};$$

$$B_5' = B_0 \frac{8(1-\varepsilon^2_{pu})+\varepsilon^4_{pu}-4(2-\varepsilon^2_{pu})\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon^2_{pu}\sqrt{1-\varepsilon^2_{pu}}};$$

$$B_1'' = B_0 \frac{2-\varepsilon^2_{pu}-2\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon^2_{pu}\sqrt{1-\varepsilon^2_{pu}}}; \qquad B_2'' = B_0 \frac{4-3\varepsilon^2_{pu}+(\varepsilon^2_{pu}-4)\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon^3_{pu}\sqrt{1-\varepsilon^2_{pu}}};$$

$$B_3'' = B_0 \frac{8(1-\varepsilon^2_{pu})+\varepsilon^4_{pu}-4(2-\varepsilon^2_{pu})\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon^4_{pu}\sqrt{1-\varepsilon^2_{pu}}};$$

$$B_4'' = B_0 \frac{16-20\varepsilon^2_{pu}+5\varepsilon^4_{pu}-(16-12\varepsilon^2_{pu}+\varepsilon^4_{pu})\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon^5_{pu}\sqrt{1-\varepsilon^2_{pu}}};$$

$$B_5'' = B_0 \frac{32-48\varepsilon^2_{pu}+18\varepsilon^4_{pu}-\varepsilon^6_{pu}-(32-32\varepsilon^2_{pu}+6\varepsilon^4_{pu})\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon^6_{pu}\sqrt{1-\varepsilon^2_{pu}}}$$

(13)

It is not difficult to show that $\lim_{\varepsilon_{pu} \to 0} B_n = 0$ for any value $n \neq 1$. For any values $\varepsilon_{pu} < 1$, $B_1 > B_2 > B_3 > B_4 > B_5$. This can easily be proved after obtaining the coefficients.

Consider the first term in the expression for the amplitude of the magnetic induction for any number of poles:

For 2p = 4

$$B_I' = B_o \frac{1-\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon_{pu}\sqrt{1-\varepsilon^2_{pu}}}; \qquad B_I'' = B_o \frac{4-3\varepsilon^2_{pu}+(\varepsilon^2_{pu}-4)\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon^2_{pu}\sqrt{1-\varepsilon^2_{pu}}};$$

For 2p = 6

$$B_I' = B_o \frac{2-\varepsilon^2_{pu}-2\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon^2_{pu}\sqrt{1-\varepsilon^2_{pu}}}; \qquad B_I'' = B_o \frac{8(1-\varepsilon^2_{pu})+\varepsilon^4_{pu}-4(2-\varepsilon^2_{pu})\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon^4_{pu}\sqrt{1-\varepsilon^2_{pu}}}$$

For 2p = 8

$$B_I' = B_o \frac{4-3\varepsilon^2_{pu}+(\varepsilon^2_{pu}-4)\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon^2_{pu}\sqrt{1-\varepsilon^2_{pu}}};$$

$$B_I'' = B_o \frac{16-20\varepsilon^2_{pu}+5\varepsilon^4_{pu}-(16-12\varepsilon^2_{pu}+\varepsilon^4_{pu})\sqrt{1-\varepsilon^2_{pu}}}{\varepsilon^5_{pu}\sqrt{1-\varepsilon^2_{pu}}}$$

DISCUSSION OF THE RESULTS

It can be seen that

$$B'_{1,2p=4} = B'_{2,2p=2}; \quad B''_{1,2p=4} = B''_{2,2p=2}; \quad B'_{1,2p=6} = B'_{3,2p=2}; \quad B''_{1,2p=6} = B''_{3,2p=2};$$
$$B'_{1,2p=8} = B'_{4,2p=2}; \quad B''_{1,2p=8} = B''_{4,2p=2};$$

Hence on increasing the number of poles of the machine the amplitude of harmonic components of magnetic induction which have the same frequency of rotation relative to the stator decrease, i.e. the uni-directional magnetic pulling force (which is proportional to the amplitude of the first harmonic of the magnetic induction) and parasitic torques (proportional to the amplitudes of the higher harmonics) decrease at increasing the number of poles.

As seen from equation (13), the amplitude of magnetic induction sharply increases at increasing the relative eccentricity. Hence the bigger the airgap the smaller the relative eccentricity, non-uniformity of the field in the airgap, additional torques, losses and load at the bearings.

Calculations on a motor rated 256kW water cooled in the stator showed that increasing the airgap 2 times (from $\delta = 1$ mm to $\delta = 2$ mm) the uni-directional magnetic pulling force in the rotor at cold state decreased by 3 times while in the heated state 4 - 5 times depending on the load and subsequently on the temperature of the motor.

CONCLUSION

The paper has successfully shown the effect of eccentricity in asynchronous machines. A uni-directional force is developed due to non-uniformity of the magnetic induction in the airgap. The force causes an extra stress to the machine and inevitably reduces its useful life. It is also clear that the magnitudes of the additional frictional torques and losses depend on the number of poles of the machine and on the extent of the eccentricity of the rotor. Furthermore, it has been established that in order to decrease the damage due to eccentricity the number of poles of the machine should be increased.

NOMENCLATURE

$a_o, c_o, a_n, b_n, c_n, d_n$	Fourier coefficients
B	magnetic induction [Wb/m^2]
p	number of pole pairs
R	radius of the stator bore [mm]
R	radius of the rotor [mm]

Greek symbols

δ	airgap length [mm]
ε	eccentricity [mm]
φ	angle of orientation [radians]
λ	magnetic conductance of the air gap
ρ	difference between the airgap length and the radius of the stator bore [mm] when the rotor is displaced by a value that shows eccentricity
ω	angular frequency [rad/s]

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