DERIVATION OF AN EXPRESSION THAT DEFINES THE PULLING FORCE IN RELAYS WITH SMALL OPERATING GAPS

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This paper derives an expression that defines magnitude of the pulling force developed in relays with small operating gaps, that is, when the length of the operating gap in between the yoke and the armature is small as compared to the area of the poles at the yoke. To obtain the expression, saturation of the core material is not considered. Results from the expression are then compared with those obtained experimentally. The comparison is found to deliver acceptable accuracy.

Keywords: Pulling force, small gap, operating gap, reluctance.

INTRODUCTION

Operating gaps in relays are defined as large or small by considering the ratio of the effective operating airgap to the area of the poles. Because of the magnetic properties of the materials in the airgaps and the poles, a majority of available electromagnetic relays have large operating gaps. An expression that defines the pulling force developed in such relays is known [Atabekov, 1960, Van Warrington, 1968]. This paper develops an expression for pulling forces in relays with small operating gaps.

DEVELOPMENT OF THE EXPRESSION

When the length of the operating gap is small as compared to the area of the poles of the yoke and when the core material is not saturated it is possible to take the following expression [Russel, 1980]:

$$ G_\delta = \frac{S}{\delta} \mu_0 \frac{dG_\delta}{d\delta} = \frac{S}{\delta^2} \mu_0 = \frac{G_\delta}{\mu_0 S} [H] $$

(1)

where

- $G_\delta$ Permeance of operating airgap dependent on displacement angle of armature
- $S$ Cross-sectional area of each plane in $m^2$
- $\delta$ Airgap length in metres
- $\mu_0$ Permeability of air

In this case for pulling force the following expression is obtained

$$ F = \frac{(IN)^2}{2G_\delta^2} \frac{G_\delta S}{\delta^2} \frac{\mu_0}{2 \mu_0 S} = \left(\frac{IN}{2} G_M^* \frac{S}{\delta^2} \right) [N] $$

(2)

where

- $F$ Pulling force
- $G_M$ Permeance of relay magnet
- $IN$ Magneto-motive force, mmf, (or ampere-turns)

The common permeance, for negligible effect of leakage fluxes, is:

$$ G_M = \frac{1}{R_M} = \frac{1}{R_\pi + R_\delta} [H] $$

(3)

where

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\[ R_m \] Reluctance of relay

\[ R_g \] Reluctance of the relay operating airgap dependent on displacement angle of armature

\[ R_x \] Reluctance of the remaining parts of the active material (core, casing, base, armature, joints, pins) not dependent on displacement angle

Substituting in expression (2) instead of \( G_m \) its value results into:

\[ F = \frac{(IN)^2}{2 \mu_o S (R_x + R_g)} = \frac{B^2 S}{2 \mu_o} [N] \] (4)

where

\[ B \] Magnetic induction

When the armature is fully attracted \( (R_g = 0) \) the pulling force becomes:

\[ F_{\Phi_o} = \frac{(IN)^2}{2 \mu_o S R_g} = \frac{\Phi_o}{2 \mu_o} [N] \] (5)

where

\[ F_{\Phi_o} \] Pulling force when the armature is fully attracted \( (R_g = 0) \)

\[ \Phi_o \] Magnetic flux when the armature is fully attracted \( (R_g = 0) \)

The relationship of magnetic fluxes at unattracted and fully attracted positions of the armature is given by:

\[ \frac{\Phi}{\Phi_o} = \frac{R_g}{R_x} = \frac{\mu_o S R_g}{\mu_o S R_x + \delta} \] (6)

where

\[ \Phi \] Magnetic flux

from where

\[ \Phi = \Phi_o \frac{\mu_o S R_g}{\mu_o S R_x + \delta} \text{[Wb]} \] (7)

Substituting in equation (4) the value of \( \Phi \) results into:

\[ F = \frac{\Phi_o (\mu_o S R_g)^2}{2 \mu_o S (\mu_o S R_x + \delta)^2} = F_{\Phi_o} (\frac{(\mu_o S R_g)^2}{(\mu_o S R_x + \delta)^2}) \] [N] (8)

If the effect of leakage fluxes is taken into account and neglect the reluctance of the soft iron of the core \( (R_m = 0) \) then in agreement with the equivalent circuit of the magnetic system of the relay [Kadete et al, 1991] it is possible to obtain the following expression:

\[ \Phi_g (R_g + R_x) = \Phi_o \frac{R_g (R_g + R_x)}{R_g + R_x + R_x} \] (9)

where

\[ R_g \] Reluctance of airgap

\[ \Phi_o \] Magnetic flux at distance

from where

\[ \Phi_g = \frac{\Phi_o R_g}{R_g + R_x + \delta} \text{[Wb]} \] (10)

Substituting in equation (4) instead of \( \Phi = \Phi_g \) the following expression is obtained:

\[ F = \frac{\Phi^2 (\mu_o S R_g)^2}{2 \mu_o S (\mu_o S (R_x + R_g) + \delta)^2} [N] \] (11)

Hence in case of an optimum relationship of reluctances of individual parts of the core, the expression for pulling force will be of the following form:

\[ F_{opt} = \frac{(IN)}{8 \mu_o S R_g} = \frac{(IN)^2 S}{8 \delta^2 \mu_o} [N] \] (12)

where

\[ F_{opt} \] Optimum pulling force

**RESULTS**

The following results were obtained and considered in order to establish the validity of the expression.
Load characteristic

Load characteristic is the relationship between pulling force of the relay and magnetomotive force, mmf, at constant value of the operating airgap. If saturation of the core and the effect of leakage fluxes are neglected the pulling force is given, in agreement with equation (12), by:

\[
F = \frac{(IN)^2 S}{8 \delta^2} \mu_o [N]
\]  

Figure 1 Load characteristic of a drop-type relay of medium power – continuous line and (b) a solenoid type relay of medium power – dotted line

Electromechanical characteristic

Electromechanical characteristic of the relay defines the relationship between pulling force of the armature and the length of airgap at constant mmf. From equation (12) it follows that at no saturation of the steel and a relatively small operating gap the value of pulling force at constant mmf may be considered inversely proportional to the square of the length of operating gap. If there is a change of the value and form of pole endings and armature, it is possible to change the electromechanical characteristic. Figure 2 shows a series of electromechanical characteristic of medium size drop type relay obtained at different values of mmf. If there is an increase of the mmf then at some value, \((IN)_{op}\), the pulling force remains equal to the opposing mechanical force (at point c) while when further increasing the mmf the armature of the relay starts to move. The value of the mmf \((IN)_{op}\) at this point is the operating mmf of the relay.

DISCUSSION OF THE RESULTS

The relay fully operates only in the case when the pulling force of the armature along its entire path of displacement is greater than the
opposing force of the mechanical load. Hence the
electromechanical characteristic corresponding to
the operating mmf of the relay exists above the
mechanical characteristic of this relay. From figure
2 it follows that operating mmf (or pulling mmf) of
the relay is obtained not by initial or ultimate loads
of the armature but by largely the loading interval
representing the mechanical characteristics. The
point of intersection, labelled b, of mechanical and
electromechanical characteristics of the relay is the
critical mmf or operating mmf. In some cases the
critical point coincides with point a, which forms
the mechanical characteristic.

In order to decrease the operating mmf and weaken
the armature's impulsive force to the core it is
desirable to match the mechanical to the
electromechanical characteristics of the relay, that
is, the angles of inclination of the characteristics
and their coordinates should be as near to each
other as is possible.

To initiate movement of the armature at release it
is necessary to decrease the mmf to a value
\((IN)_{op}\) (operating mmf at release) at which the
electromechanical (pulling) characteristic passes
through the last point, which is also part of the
mechanical characteristic.

CONCLUSION

The paper has developed an expression that
defines the pulling force in relays with small gaps.
The expression was tested through experimentation with relays at different conditions
and the results obtained coincide with those
obtained through computations using the derived
expression. The expression can hence be used to
predict the pulling force in such relays and may
prove useful to both the designers and users of the
relays.

NOMENCLATURE:

- \(B\) Magnetic induction
- \(F\) Pulling force
- \(F_{\infty}\) Pulling force when the armature is fully
attracted \((R_{g} = 0)\)
- \(F_{op}\) Optimum pulling force
- \(G_{g}\) Permeance of operating airgap
dependent on displacement angle of
armature
- \(G_{M}\) Permeance of relay magnet
- \(IN\) Magneto-motive force, mmf, (or
ampere-turns)
- \((IN)_{op}\) Operating mmf
- \((IN)_{op}\) Operating mmf at release
- \(R_{s}\) Magneto-motive force
- \(R_{g}\) Reluctance of the relay operating airgap
dependent on displacement angle of
armature
- \(R_{s}\) Reluctance of the remaining parts of the
active material (core, casing, base,
armature, joints, pins) not dependent on
displacement angle
- \(R_{g}\) Reluctance in between the soft iron
space
- \(R_{g}\) Reluctance of airgap.
- \(R_{m}\) Reluctance of relay
- \(R_{g}\) Reluctance of the gap in between base
and the armature along the motion axis
of the latter
- \(S\) Cross-sectional area of each plane in
\(m^{2}\)

Greek Symbols

- \(\delta\) Airgap length in metres
- \(\mu_{0}\) Permeability of air
- \(\Phi\) Magnetic flux
- \(\Phi_{\infty}\) Magnetic flux when the armature is
fully attracted \((R_{g} = 0)\)
- \(\Phi_{e}\) Magnetic flux at distance

REFERENCES

1. Atabekov, G.I.: *The Relay Protection of
High Voltage Networks*, Pergamon Press,
1960.
2. Kadete, H. And Chambega, D.J.: *Design,*
Construction and Experimentation of a Relay. 
Faculty of Engineering, University of Dar es Salaam.
