DERIVATION AND ANALYSIS OF MAXIMUM AND CONDITIONAL WORK OF AN ELECTROMAGNET IN RELAYS WITH SMALL GAPS AND CONSTANT ARMATURE LOAD

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This paper derives and establishes an expression for maximum and conditional work of electromagnets in relays with small operating air-gaps and whose armature load is constant. Then the expression is thoroughly analysed and its validity is established experimentally by comparing results obtained through computation using the expression with those obtained experimentally.

Keywords: *Conditional work, Maximum work, Constant load, Core.*

INTRODUCTION

The load to the armature when a relay is in operation is quite complex because the conditions of the path describing movement of the armature considerably change. In order to simplify the analysis of the displacement process, armature loads may be considered linear. Maximum and conditional work for electromagnets in relays with small gaps and with linear armature load was expressed and validated [1]. It was found that for an armature load changing linearly maximum work necessary to overcome this load equals half the maximum work performed by the relay armature. The displacement process can, however, also be simplified if the armature load is considered constant. This paper is a first attempt to also simplify the process by considering a constant armature load. The analysis involves the derivation of maximum and conditional work for relays with small gaps only.

DERIVATION OF CONDITIONAL WORK WHEN THE ARMATURE LOAD IS CONSTANT

Pulling force in relays with small gaps is given by [2]:

$$F = \frac{(IN)^2}{2\mu_o S(R_\pi + R_\delta)^2} = \frac{\phi^2}{2\mu_o S} = \frac{B^2 S}{2\mu_o} N \qquad (1)$$

where R_π - total reluctance of all elements in the magnetic circuit the value of which is independent of the position of the armature; R_δ - magnetic resistance of the operating airgap.

Magnetic flux at the end of the core in the space between the armature and the yoke can be expressed as follows [3]:

$$\phi_l = \frac{IN(R_o gl + 2)}{2[l(R_{si} + R_o R_B g) + q(R_B + R_o)]} \qquad (2)$$

Work necessary to overcome armature mechanical load (opposing force) whose value is constant and denoted by F_1 is often a conditional work and is expressed as follows [1]:

$$A_1 = F_1\delta_1 \qquad (3)$$

Graphically this work is the area of rectangle F_1 a δ_1 O (Figure 1). Substituting in the expression for A_1 the value of F_1 from equation (1) and putting the value of ϕ from equation (2) the following equation is obtained:

$$A_l = \frac{(IN)^2(R_o gl + 2)^2\delta_l}{2\mu_o S^4[(lR_{av} + R_o R_b g) + q(R_b + R_o)]^2} = \frac{\Gamma\delta_1}{(B + D\frac{\delta_1}{\mu_0 S})^2} \qquad (4)$$

where

$$\Gamma = \frac{(IN)^2(R_o gl + 2)^2}{8\mu_o S}; R_b = R_\delta + R_{ins} + R_{si}$$

$$B = l B_{av} + qR_0 + (R_{ins} + R_{si})(q + R_0 gl);$$

$$D = q + R_0 gl$$

where R_b - reluctance of the relay air-gap taking into account the fastenings attaching the core to the relay base; R_δ - reluctance of the air-gap dependent on the movement and hence position of the armature; R_{ins} - reluctance of the gap in between the armature and the core when the armature is fully attracted; the space in question usually has some non-magnetic material; R_{si} - reluctance of the gap in between base and the armature along the motion axis of the latter, this value is considered not dependent on position of the armature.

DETERMINATION OF MAXIMUM

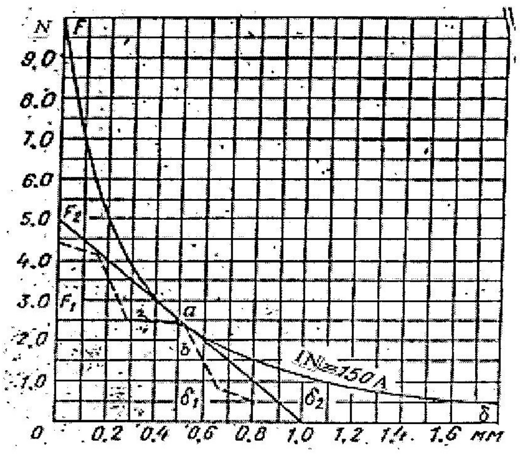


Figure 1: Curves for the determination of work for electromagnets

CONDITIONAL WORK

In order to determine the conditions at which the value of conditional work of the relay will be maximum, the derivative of A with respect to δ_1 is taken and equated to zero, the results being:

$$\frac{dA_1}{d\delta_1} = \Gamma \frac{\left(B + D\frac{\delta_1}{\mu_o S}\right)^2 - \delta_1\left(\frac{2BD}{\mu_o S} + \frac{2D^2}{\mu_o^2 S^2}\right)}{\left(B + D\frac{\delta_1}{\mu_o S}\right)^4} = 0 \tag{5}$$

from where

$$B^2 - \frac{D^2 \delta_1^2}{\mu^2_o S^2} = 0 \text{ or } B = DR_\delta \tag{6}$$

Putting back the values of B and D results into

$$R_\delta = \frac{l R_{av} + q R_o}{q + R_o gl} + R_{ins} + R_{si} \tag{7}$$

Neglecting the effect of leakage flux between core and casing of the relay $(g = 0)$ then the condition for maximum use of conditional work of the relay becomes:

$$\left.\begin{aligned} R_\delta &= l R_{av} + R_0 + R_{ins} + R_{si} \\ \text{or} \quad R_\delta &= R_\pi \end{aligned}\right\} \tag{8}$$

Substituting in expression (3) the value of F_1 from equation (1) and putting the value R_δ equal to the value of R_1 an expression for maximum value of used conditional work of the relay is obtained as follows:

$$A_{1m} = \frac{(IN)^2 \delta}{2\mu_0 S(R_\pi + R_\delta)^2} = \frac{(IN)^2 R_\delta}{2(R_\pi + R_\delta)^2} = \frac{(IN)^2}{8 R_\pi} Nm \tag{9}$$

Hence the maximum value of used conditional work (work at constant load) is four times smaller the maximum work A_m given by [3]:

$$A_m = \frac{(IN)^2}{2 R_\pi}, Nm \tag{10}$$

which may be produced by the relay.

RESULTS

If the value of R_δ is not equal to R_π then dividing the numerator by the denominator in equation (8) by R_π^2 and representing the ratio R_δ to R_π by a the following expression for conditional work is obtained:

$$A_1 = \frac{(IN)^2 a}{2 R_\pi (1 + a)^2} = A_m \frac{a}{(1 + a)^2} \tag{11}$$

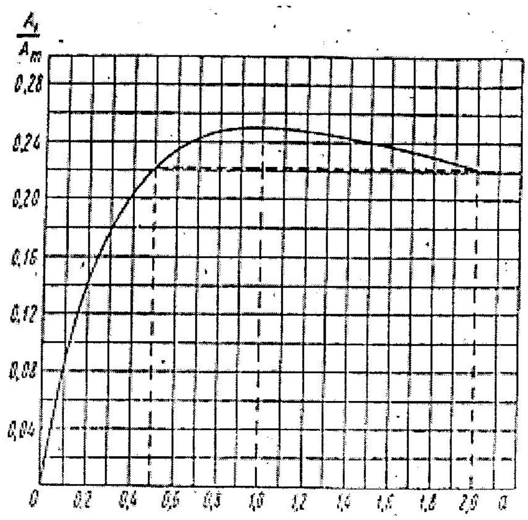


Figure 2: Curve showing the relationship between the ratio of A_1 to A_m and the value a

Figure 2 shows the relationship between the ratio A_1 to A_m and the value a. From the relationship it is clear that change in ratio R_δ to R_π within big limits from 0.5 to 2 - the value of used conditional work of the relay decreases no more than 11%.

If the value of mmf is substituted in equation (9) then the following expression is obtained:

$$A_{1m} = 1.6 \times 10^{-8} \frac{P}{CR_\pi} = 1.6 \times 10^{-8} \frac{P}{CR_\delta} \qquad (12)$$

Hence in case of uniform distribution of flux in the working inter-core space, the absence of saturation in the steel and negligible effect of flux leakages, the relationship of the value of conditional work of the relay and the input power must be linear.

Heat conductance for heat influx, emitted by the relay winding into surrounding air is given by:

$$G_\theta = \frac{P}{v} \qquad (13)$$

Electrical conductance of the winding window is given as

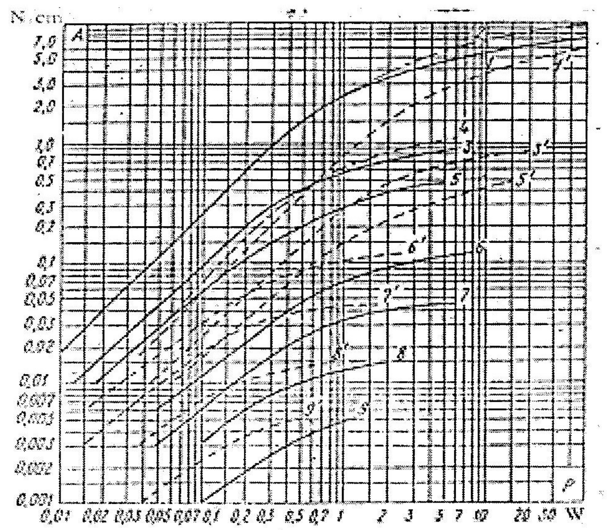
$$G_E = \frac{1}{C} \qquad (14)$$

Substituting the values of P and C obtained in equations (13) and (14) into equation (12) the following expression is obtained:

$$A_{1m} = 1.6 \times 10^{-8} vG_\delta G_E G_\theta \qquad (15)$$

Hence the working capability of the magnetic system is proportional to the temperature rise of the winding and a product of magnetic susceptance, electric conductance and heat conductance [5, 6].

Figure 3 shows experimental results of the relationship between conditional work and the value of power consumed for seven relays drop type of different sizes at nominal values of armature move (gap). The basic data for these relays is given in Table 1. In order to neglect the effect of the value of the ratio $\frac{l}{d}$ on the power consumed the curves of conditional work of the last four types of relays were computed and



Continuous line – Experimental
Dotted line – computed
1 – Relay No.1 (d_p = 30 mm);
2 – Relay No. 1 (d_p = 18 mm);
3 – Relay No. 2 (d_p = 15 mm);
4 – Relay No. 2 (d_p = 9 mm);
5 – Relay No. 3;
6 – Relay No. 4;
7 – Relay No. 5 (d_p = 3 mm);
8 – Relay No. 6 (d_p = 2.4 mm);
9 – Relay No. 7 (d_p = 2 mm).

Figure 3: Curves showing the relationship between conditional work of a relay and the consumed power

Table 1: Parameters of the relays used for the investigation.

No	Diameter of core d, mm	Diameter of Pole d_p, mm	Length of core l, mm	l/d	$Cx10^{-6}$ Ω	Move of Armature (Gap) δ,mm	Mass of Relay Q,gr
1	18	30	140	7.8	1.7	1.6	2300
2	9	15	70	7.8	3.8	0.8	290
3	7	11	55	9	5.1	1.1	112
4	5	8	15	3.0	23.4	0.4	17
5	3	5	10	3.3	39	0.45	6
6	2.4	3.8	7.5	3.1	40	0.4	3.1
7	2	3	6.5	3.2	46	0.4	1.8

referenced to the value $\frac{l}{d} = 7.8$ while the curves of the first three types of relays to the value $\frac{l}{d} = 3.2$.

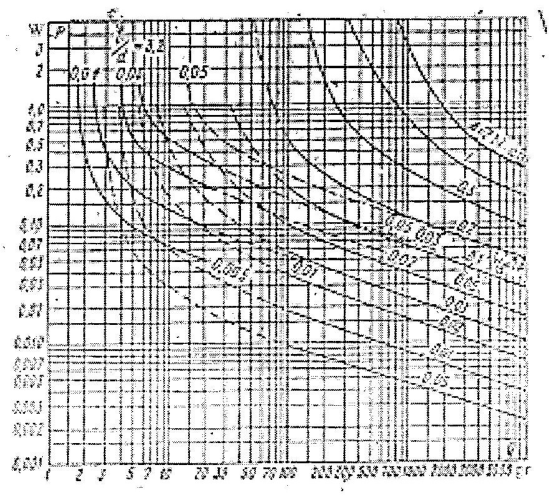


Figure 4: Curves showing the relationship between power consumed by the relay and its mass at different values of conditional work

The computed curves are shown as broken lines in Figure 4.

The relationship between power consumed by the drop type relays and their mass at different values of conditional work is shown in Figure 4.

DISCUSSION OF THE RESULTS

From the results shown in Figure 3, it follows that only the initial portion of the curves of the first three types of relays is linear.

Reluctance of the active material depends on the value of magnetic induction of the steel and therefore the value of optimum move of the relay armature and the most convenient area of the pole tips will change with the relay load.

CONCLUSION

The paper has established that the maximum value of used conditional work needed in relays with small operating gaps to overcome an armature load which is constant is four times smaller than the maximum work which may be produced by the relay. Further, for a given ampere-turns the maximum work the relay can develop is proportional to half the reciprocal of the total reluctance of all the elements in the magnetic circuit the value of which is independent of the armature position. It has also shown that in case of uniform distribution of flux in the working inter-core space, the absence of saturation in the steel and the neglect of the effect of flux leakages the relationship of the value of conditional work of the relay and the input power must be linear.

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