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Comparative Analysis of Multiplicative and Additive Noise Based Automated Regularizations in Non-Linear Diffusion Image Reconstruction

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ABSTRACT

Multiplicative and additive noises are often introduced in image signals during the image acquisition process and result into degradation of image features. The work done by Perona and Malik in 1990 and its modified versions revolutionized the way through which noises or speckles are removed. The Perona-Malik model requires tuning of the regularization parameter to control and prevent staircase artifacts in restored images. The current manual tuning is a challenging and time consuming practice when a long queue of images is registered for processing. Attempt to automate the regularization parameter appeared in Perona-Malik model with self-adjusting shape-defining constant. Although both multiplicative and additive noise based automated regularizations were presented, the paper stayed silent on matters concerning the automation method that fits with speckle reduction. This paper therefore, presents a comparative analysis of additive and multiplicative noise based automated regularizations. Simulation results and paired samples T-tests reveal that the multiplicative noise based automation outperforms the additive noise based automation for small speckle variances. However, the two automation methods do not significantly differ when large speckle variances are assumed.

Keywords: Additive Noise, Image Processing, Multiplicative Noise, Non-Linear Diffusion, Regularization.

INTRODUCTION

In image acquisition process, coherent or in-phase waves are usually projected towards the target object. Depending on imagery mechanisms, different types of waves are used in capturing the image. For example, coherent radiation of micro electromagnetic waves is used in synthetic aperture radar imaging, high energy electromagnetic waves are used in X-ray imaging, coherent light waves are used in laser imaging, and acoustic or sound waves are used in ultrasound imaging (Raney, 1998; Huang *et al.*, 2009; Liu *et al.*, 2013; Bharathi *et al.*, 2014; Adabi *et al.*, 2017; Kessy *et al.*, 2017a). When the

transmitted waves reach the target object, they are reflected back to an active sensor for image reconstruction (Chen *et al.*, 2019). Due to the variation and in homogeneity of object characteristics and dimensions, these waves are often back scattered into multipath components that travel different distances to reach the sensor location. Most often, the back scattered waves interfere constructively or destructively generating speckles characterized by black and light spots on images (Kessy *et al.*, 2017a). Speckles significantly undermine the quality and usefulness of images because they corrupt and hide textures and edges that are crucial for accurate assessment of the captured

scene or extraction and recognition of features and patterns from images (Meenakshi and Punitham, 2011).

Attempts to generate speckle free images came up with different type of filters whereby the image pixel is replaced by an estimated value. During the emerging age of image processing, spatial or linear filters were proposed to approximate pixels' values based on local statistics of the image. These filters include, but not limited to, the Frost filter that replaces the central pixel by a weighted sum of neighboring pixels, Lee and Wiener filters that smoothen the image on variance basis, and Gamma Map filter that estimates the pixel value based on Gamma estimation of contrast ratios (Mansourpour *et al.*, 2000; Chopra and Anand, 2014; Jaybhay and Shastri, 2015). Despite the merits of linear filters in generating speckle free images, they are associated with high degree of blurs and distortions of textures and edges (Jaybhay and Shastri, 2015).

To address this weakness of linear filters, Perona and Malik proposed a nonlinear diffusion filter to smoothen internal regions of the image while fleeing regions where sharp contrast variations or edges are detected (Perona and Malik, 1990). The Perona-Malik model significantly attracted scholars' attention due to its edge preservation capabilities. This attraction is characterized by several modifications of the Perona-Malik model that are published in different journals (Guo *et al.*, 2012; Kessy *et al.*, 2017a; Kessy *et al.*, 2017b; Maiseli *et al.*, 2018). These works are establishing stable and accurate models that deal with different noise variants and staircase artifacts caused by the ill-posed aspect associated with the partial differentiation applied in the Perona-Malik kernel (Liu *et al.*, 2013; Jain and Ray, 2019; Yao *et al.*, 2019). In general, the Perona-Malik model is made of a diffusion kernel functional that approximates the pixel value and the regularization term,

which has been added to control the ill-posed aspect of the model and prevents staircase artifacts in the despeckled image.

The classical regularization requires manual tuning of the Lagrange multiplier or regularization parameter. This manual tuning consumes time and is harmful in domains that operate under high workload and high-level of accuracy (Maiseli *et al.*, 2018). The regularization parameter should be correctly chosen so that a proper image can be recovered (Liu *et al.*, 2013). The study that automates the regularization parameter is presented in Perona-Malik model with self-adjusting shape-defining constant whereby automation analysis based on both additive and multiplicative noise models came up with two distinct automated regularization parameter formulae (Maiseli *et al.*, 2018). However, authors did not present a formulation that fits with speckle granularities and proprieties. Therefore, this paper presents a comparative analysis of multiplicative and additive noise based automated regularizations that establish a general agreement for automated regularization in speckles reduction process.

MATERIALS AND METHODS

Modified Perona-Malik Models

The Perona-Malik filter was modelled based on the concept of anisotropic diffusion as presented by Fick's law (Perona and Malik, 1990; Paul and Laurila, 2014). The idea behind the modelling was to smoothen internal regions of the image while fleeing regions with sharp contrast variations or edges (Perona and Malik, 1990; Maiseli *et al.*, 2018). The fleeing of regions was achieved through adaptive diffusion under assumption that stronger smoothing is needed in areas with large diffusivity value and vice versa. A high norm of the gradient is measured in area with smaller

diffusivity value, which predicts the location of edges in an image and hence no or less smoothing should be applied. The Perona-Malik model incorporates the diffusion Kernel functional that approximates the pixel value and the regularization term that controls the ill-posed aspect of the model and prevents staircase artifacts in the despeckled image. Therefore, modifications of the

Perona-Malik model mainly focus on the manipulation of either diffusion Kernel functional or the regularization term. Different evolution equations were derived with convincing results including, but not limited to, the evolution equations in equations (1), (2), (3) and (4) (Kessy *et al.*, 2017a; Kessy *et al.*, 2017b; Maiseli *et al.*, 2018).

$$\frac{\partial u}{\partial t} = \operatorname{div} \left(\frac{1}{1 + \left(\frac{|\nabla u|}{k}\right)^2} \nabla u \right) - \lambda(u - f) \dots\dots\dots (1)$$

$$\frac{\partial u}{\partial t} = \operatorname{div} \left(\frac{1}{1 + \left(\frac{|\nabla u|}{k}\right)^2} \nabla u \right) - \lambda \left(1 - \frac{f}{u}\right) \dots\dots\dots (2)$$

$$\frac{\partial u}{\partial t} = \operatorname{div} \left(\frac{1}{1 + \left(\frac{|\nabla u|}{k}\right)^2} \nabla u \right) - \frac{\lambda(u-f)}{u^2} \dots\dots\dots (3)$$

$$\frac{\partial u}{\partial t} = \operatorname{div} \left(\frac{1}{\sqrt{1 + \left(\frac{|\nabla u|}{k}\right)^2}} \nabla u \right) - \frac{\lambda(u-f)}{u^2} \dots\dots\dots (4)$$

$$(x, t) \in \Omega \times (0, T)$$

$$u(x, 0) = f, x \in \Omega \dots\dots\dots (5)$$

$$\frac{\partial u}{\partial n} = 0, (x, t) \in \partial\Omega \times (0, T) \dots\dots\dots (6)$$

The function u represents the speckle free image, f represents the noisy image, div denotes the divergence, ∇ denotes the gradient, k symbolizes the shape-defining constant. Also, the supporting domain is represented by Ω in which (x, t) captures the despeckled image in spatial and time domain.

Automation of the Regularization Parameter

Lagrange multiplier or regularization parameter enables the adjustment between the despeckled image and the noisy image during a specific iteration. This parameter is tuned to control and prevent staircase artifacts in the restored image and therefore, different images may require different values. The work done by Maiseli *et al.* (2018) proposed an automated regularization parameter based on the local information of an image. Authors assumed an optimal solution for the evolution equation (1) such that $\frac{\partial u}{\partial t} \rightarrow 0$ for $t \rightarrow \infty$, and the resulting equation was multiplied by $(u - f)$, as given by equation (7).

Equation (7) was then modified by assuming that the noise variance is known so as to obtain equation (8) or the automated value of λ .

$$0 = (u - f) \operatorname{div} \left(\frac{1}{1 + \left(\frac{|\nabla u|}{k}\right)^2} \nabla u \right) - \lambda (u - f)^2 \dots\dots\dots (7)$$

$$\lambda = \frac{1}{|\Omega| \times \delta_a^2} \int_{\Omega} (u - f) \operatorname{div} \left(\frac{1}{1 + \left(\frac{|\nabla u|}{k}\right)^2} \nabla u \right) dx, \dots\dots\dots (8)$$

where δ_a^2 represents the variance of a zero mean additive noise as given by equations (10). It should be noted that equation (9) represents the mean value formulation of the additive noise.

$$\frac{1}{|\Omega|} \int_{\Omega} n dx = 0. \dots\dots\dots (9)$$

$$\lambda = \frac{1}{|\Omega| \times \delta_m^2} \int_{\Omega} \left(1 - \frac{f}{u}\right) \operatorname{div} \left(\frac{1}{1 + \left(\frac{|\nabla u|}{k}\right)^2} \nabla u \right) dx, \dots\dots\dots (11)$$

where δ_m^2 represents the variance of the multiplicative noise as given by equation (13). Equation (12) provides the mean value formulation of the multiplicative noise.

$$\frac{1}{|\Omega|} \int_{\Omega} n^2 dx = \delta_a^2 \dots\dots\dots (10)$$

Accordingly, equation (2) was used to derive the automated parameter λ , as given by equation (11).

$$\frac{1}{|\Omega|} \int_{\Omega} n dx = 1 \dots\dots\dots (12)$$

$$\frac{1}{|\Omega|} \int_{\Omega} (n - 1)^2 dx = \delta_m^2 \dots\dots\dots (13)$$

RESULTS AND DISCUSSION

The additive and multiplicative noise based automated regularization parameters were incorporated subsequently in all the evolution equations presented in equations (1), (2), (3) and (4). Simulations were run for the peak signal to noise ratio (PSNR) and the structure similarity index (SSIM) to measure the noise removal and the feature preservation capabilities, respectively (Kessy *et al.*, 2017a; Maiseli *et al.*, 2018). The shape-defining constant (K) was set to 1.96 where 1000 iterations were performed for each noise level. Also, a 300 by 300-pixel size synthetic image was used in simulations. This image was used for all the methods based on the fact that the paired samples T-test imposes the

evaluation of objects under the same condition.

Table 1 presents PSNR values obtained from several experiments using MATLAB image processing tool box with small scaled noise variances. Pair 1, Pair 2, Pair 3 and Pair 4 represent PSNR variations with noise variances for equations (1), (2), (3) and (4) automated with the additive noise based automation (AA) and multiplicative noise based automation (MA). To perform comparisons for further understating of the noise removal capability of the automation methods, statistical summaries were established as in Table 2. It was observed that the mean of MA leads in all the table entries and hence it deserves superior considerations.

Table 1: PSNR for small scaled speckle variances

Small scaled variance	PSNR									
	Pair 1		Pair 2		Pair 3		Pair 4		SumPSNR	
	AA	MA	AA	MA	AA	MA	AA	MA	AA	MA
0.1	18.5425	22.6123	22.0450	30.1008	21.0624	30.4488	18.9008	27.6904	80.55074	110.8524
0.2	18.9202	22.8841	30.2450	30.1709	24.2719	30.3520	30.1239	29.7461	103.5610	113.1532
0.3	20.5696	30.2261	25.4945	30.0594	19.6592	29.4966	29.5660	29.3836	95.28932	119.1658
0.4	20.402	30.1302	29.6966	30.2961	28.6797	30.1043	23.2274	29.6318	102.0058	120.1625
0.5	18.5455	30.1162	29.0758	29.9763	30.7103	29.7688	29.4414	29.1111	107.7732	118.9724
0.6	20.0194	27.9052	30.1403	30.2454	31.0339	30.5161	29.9070	29.5448	111.1007	118.2116
0.7	21.2470	30.0090	30.0784	30.0631	28.9393	29.9165	24.1295	29.4717	104.3942	119.4604
0.8	18.4676	30.0649	30.1816	30.2190	21.1227	29.5382	26.4952	29.4213	96.26721	119.2435
0.9	21.3050	30.0867	30.0583	30.0620	28.0787	30.1121	26.1094	29.1719	105.5514	119.4328
1	22.0387	30.1865	30.1034	30.2197	30.6818	29.9381	29.9743	29.4846	112.7983	119.8290

Table 2: Paired samples statistics of PSNR for small scaled speckle variances

Automated Pair	Mean	Std. Deviation
Pair 1	MA	28.4221
	AA	20.0057
Pair 2	MA	30.1412
	AA	28.7118
Pair 3	MA	30.0191
	AA	26.4240
Pair 4	MA	29.2657
	AA	26.7875
Sum SSIM	MA	117.8484
	AA	101.9292

Furthermore, to establish a proper conclusion about this outperformance, paired samples T-tests were used to confirm whether the observed means significantly differ. From Table 3 in Pair 1 entry, the Paired samples T-test $t(9) = 10.0870$ reveals that the means 28.4221 ± 3.0701 and 20.0057 ± 1.3188 significantly differ at 95% confidence interval of difference with a p value or Sig.(2-tailed) of $0.000 < 0.05$. This observation validates the outperformance of the multiplicative noise based automation over the additive noise based automation when the evolution equation (1) is assumed. The

outperformance was also validated for evolution equation (3) and evaluation equation (4) giving p values of 0.0290 and 0.0470, respectively, which are less than 0.05. Unlike this three entries, the means for evolution equation (2) do not significantly differ because the p value was $0.131 > 0.05$. The general agreement is that the multiplicative noise based automation should be considered because, despite its high mean values, the sum PSNR's means significantly differ with p value of $0.0000 < 0.05$.

Table 3: Paired samples tests of PSNR for small scaled speckle variances

		Paired Differences				t	Sig. (2-tailed)
		Mean	Std. Deviation	95% Confidence Interval of the Difference			
				Lower	Upper		
Pair 1	MA - AA	8.4163	2.6384	6.5289	10.3037	10.0870	0.0000
Pair 2	MA - AA	1.4293	2.7183	-0.5152	3.3739	1.6630	0.1310
Pair 3	MA - AA	3.5951	4.3747	0.4656	6.7246	2.5990	0.0290
Pair 4	MA - AA	2.4782	3.3992	0.0465	4.9099	2.3050	0.0470
Sum PSNR	MA - AA	15.9191	7.8085	10.3332	21.5050	6.4470	0.0000

The two automation methods behaved differently depending on the considered noise variance. Table 4 presents the raw data of PSNR obtained when large scaled noise variances were assumed. Also, statistical analysis was done to establish

fair comparisons between the PSNRs for additive and multiplicative based automation methods when an image with large scaled speckle variances is assumed, as in Tables 5 and 6.

Table 4: PSNR for large scaled speckle variances

Large scaled variances	PSNR									
	Pair 1		Pair 2		Pair 3		Pair 4		Sum PSNR	
	AA	MA	AA	MA	AA	MA	AA	MA	AA	MA
10	27.0330	30.1844	30.2344	30.2496	29.7193	30.0672	29.0594	29.2341	116.0462	119.7354
20	20.6301	30.2413	30.0632	30.0637	29.5610	30.2809	30.0694	29.5148	110.3239	120.1009
30	30.2227	30.0640	30.2154	30.2187	30.2977	30.1583	29.4830	29.4170	120.2189	119.8582
40	27.3743	30.2173	30.0657	30.0633	29.833	30.0263	29.7131	29.3120	116.9862	119.6191
50	30.0132	30.0643	30.2258	30.2262	29.9713	30.1898	28.7070	29.1189	118.9175	119.5993
60	28.0792	30.2191	30.2719	30.2699	29.9109	30.0761	29.5003	29.3179	117.7624	119.8832
70	30.3061	30.2139	30.2492	30.2505	30.2476	30.1007	29.3327	29.5043	120.1357	120.0695
80	30.3305	30.2502	30.1721	30.1687	30.2563	30.0607	29.27084	29.3134	120.0298	119.7931
90	30.4348	30.4207	30.0252	30.0251	30.1577	30.4040	29.2433	29.3128	119.8611	120.1627
100	30.0383	30.0256	30.4205	30.4221	30.2574	30.3356	29.4211	29.3401	120.1375	120.1234

The same as for small scaled noise variances, the multiplicative noise based automation presents high mean values in most of the entries for large scaled variances as in Table 5. The application of paired samples T-tests reveals that the outperformance of the multiplicative noise based automation is not significant for large scaled variance under all the

evolution equations. This is because all the *p* values or Sig. (2-tailed) in Table 6 are not less than 0.05. For example, Paired samples T-test $t(9) = 1.7990$ reveals that the means 30.1900 ± 0.1156 and 28.4462 ± 3.0456 do not significantly differ at 95% confidence interval of difference with a *p* value or Sig.(2-tailed) of $0.1060 > 0.05$.

Table 5: Paired samples statistics of PSNR for large scaled speckle variances

Automated Pair		Mean	Std. Deviation
Pair 1	MA	30.1900	0.1156
	AA	28.4462	3.0456
Pair 2	MA	30.1958	0.1196
	AA	30.1943	0.1183
Pair 3	MA	30.0212	0.2604
	AA	30.1958	0.1196
Pair 4	MA	29.3385	0.1184
	AA	29.3800	0.3653
Sum SSIM	MA	119.8945	0.2102
	AA	118.0419	3.0963

Table 6: Paired samples tests of PSNR for large scaled speckle variances

		Paired Differences				t	Sig. (2-tailed)
		Mean	Std. Deviation	95% Confidence Interval of the Difference			
				Lower	Upper		
Pair 1	MA - AA	1.7438	3.0659	-0.4493	3.9371	1.7990	0.1060
Pair 2	MA - AA	0.0014	0.0052	-0.0023	0.0052	0.8690	0.4080
Pair 3	AA - MA	-0.1745	0.2437	-0.3489	-0.0001	-2.2650	0.0500
Pair 4	MA - AA	-0.0414	0.2851	-0.2454	0.1624	-0.4600	0.6560
Sum PSNR	MA - AA	1.8525	3.1101	-0.3722	4.0774	1.8840	0.0920

The structure similarity index denoted as SSIM, measures the edges or textures recovery capability for several despeckling methods. The higher the SSIM value the better the performance in recovering useful features of the image. Like for the PSNR, Table 7 presents the raw data of SSIM obtained from several experiments in which small scaled noise variances were manipulated.

For comparisons purposes, statistical summaries were established whereby it was observed that the multiplicative noise based automation (MA) presents higher mean values compared to the additive noise based automation, as in Table 8. The same as for PSNR, paired samples T-tests

were used to establish a proper conclusion about the difference between mean values of SSIM for the two automation methods. From Table 9 in Pair 1 and Pair 3 entries, the outperformance of the multiplicative based automation is validated by the T-tests because the p values of 0.000 and 0.0360 are less than 0.05. In contrast, p values of Pair 2 and 4 are greater than 0.05 and hence the differences are not significant in these two cases. In light of the high mean values in most of the entries and the significance of difference in the sum SSIM entry, the multiplicative noise based automation should be used for more texture and edges recovery when small scaled variances are assumed.

Table 7: SSIM for small scaled speckle variances

Small scaled variances	SSIM									
	Pair 1		Pair 2		Pair 3		Pair 4		Sum SSIM	
	AA	MA	AA	MA	AA	MA	AA	MA	AA	MA
0.1	0.1326	0.5623	0.4506	0.9512	0.8858	0.9514	0.2127	0.8835	1.6817	3.3484
0.2	0.1570	0.5722	0.9753	0.9518	0.6315	0.9519	0.8805	0.8849	2.6443	3.3608
0.3	0.2282	0.9650	0.7442	0.9508	0.6822	0.9509	0.8844	0.8843	2.5390	3.7511
0.4	0.2464	0.9852	0.9684	0.9584	0.9312	0.9522	0.8693	0.8846	3.0155	3.7805
0.5	0.1388	0.9852	0.8871	0.9512	0.9512	0.9511	0.8833	0.8833	2.8605	3.7709
0.6	0.2111	0.8154	0.9571	0.9511	0.9509	0.9510	0.8841	0.8844	3.0033	3.6019
0.7	0.3248	0.9849	0.9534	0.9510	0.9314	0.9508	0.8536	0.8847	3.0632	3.7715
0.8	0.0128	0.9522	0.9548	0.9509	0.7029	0.9513	0.8694	0.8851	2.5400	3.7395
0.9	0.3528	0.9528	0.9564	0.952	0.9127	0.9517	0.8624	0.8844	3.0843	3.7410
1	0.4068	0.9538	0.9526	0.9513	0.9463	0.9514	0.8766	0.8844	3.1824	3.7410

Table 8: Paired samples statistics of SSIM for small scaled speckle variances

Automated Pair	Mean	Std. Deviation
Pair 1	MA	0.8729
	AA	0.2211
Pair 2	MA	0.9520
	AA	0.8800
Pair 3	MA	0.9514
	AA	0.8526
Pair 4	MA	0.8844
	AA	0.8077
Sum SSIM	MA	3.6607
	AA	2.7615

In terms of SSIM, the two automation methods also behaved differently on noise variance basis. Table 10 presents the raw data of SSIM obtained when large scaled noise variances were assumed. Also, for fair comparisons between the automation methods, statistical analysis was used as in Table 11 and 12. It was observed in Table 11 that the multiplicative noise based automation presents high mean values in most of the entries for large scaled variances. To establish conclusions about

this outperformance, significance tests were performed as in Table 12.

From Table 12, it was observed that all the p values represented by Sig.(2-tailed) are greater than 0.05, which means that there is no significant difference between the results from multiplicative and additive noise based automations. This aspect indicates that any automation method can be used when large scaled noise variances are assumed.

Table 9: Paired Samples Tests of SSIM for Small scaled speckle Variances

		Paired Differences				t	Sig. (2-tailed)
		Mean	Std. Deviation	95% Confidence Interval of the Difference			
				Lower	Upper		
Pair 1	MA - AA	0.6517	0.1688	0.5309	0.7725	12.2060	.0000
Pair 2	MA - AA	0.0719	0.1654	-0.0463	0.1903	1.3760	.2020
Pair 3	MA - AA	0.0987	0.1272	0.0077	0.1897	2.4550	.0360
Pair 4	MA - AA	0.0767	0.2089	-0.0727	0.2262	1.1601	.2750
Sum SSIM	MA - AA	0.8992	0.3543	0.6457	1.1527	8.0250	.0000

Table 10: SSIM for Large-scaled Speckle Variances

Large scaled variance	SSIM									
	Pair 1		Pair 2		Pair 3		Pair 4		Sum SSIM	
	AA	MA	AA	MA	AA	MA	AA	MA	AA	MA
10	0.7485	0.9523	0.9506	0.9511	0.95203	0.9521	0.8846	0.8847	3.5358	3.7403
20	0.4621	0.9850	0.9510	0.9510	0.9518	0.9519	0.8848	0.8849	3.2497	3.7729
30	0.9787	0.9509	0.9507	0.9509	0.9514	0.9513	0.8844	0.8844	3.7653	3.7377
40	0.7822	0.9509	0.9520	0.9519	0.9506	0.9506	0.8830	0.8830	3.5678	3.7365
50	0.9638	0.9519	0.9514	0.9514	0.9525	0.9526	0.8836	0.8837	3.7514	3.7396
60	0.8356	0.9511	0.952218	0.9520	0.9514	0.9514	0.8843	0.8843	3.6236	3.73897
70	0.9582	0.9521	0.95147	0.9516	0.9515	0.9514	0.8850	0.8850	3.7461	3.7402
80	0.96	0.9516	0.951068	0.9509	0.9519	0.9519	0.8846	0.8846	3.7476	3.7391
90	0.9581	0.9522	0.950771	0.9506	0.9517	0.9518	0.8843	0.8843	3.7450	3.7390
100	0.9523	0.9508	0.952115	0.9521	0.9517	0.9518	0.8848	0.8848	3.7410	3.7395

Table 11: Paired Samples Statistics of SSIM for Large-scaled Speckle Variances

Automated Pair	Mean	Std. Deviation
Pair 1	MA	0.9548
	AA	0.8599
Pair 2	MA	0.9514
	AA	0.9513
Pair 3	MA	0.9517
	AA	0.9517
Pair 4	MA	0.8844
	AA	0.8843
Sum SSIM	MA	3.7424
	AA	3.6473

Table 12: Paired Samples test of SSIM for Large-scaled Speckle Variances

		Paired Differences				t	Sig. (2-tailed)
		Mean	Std. Deviation	95% Confidence Interval of the Difference			
				Lower	Upper		
Pair 1	MA – AA	0.0949	0.1724	-0.0284	0.2183	1.7410	0.1160
Pair 2	MA – AA	0.0000	0.0001	-0.0000	0.0001	0.7230	0.4880
Pair 3	MA – AA	0.0000	0.0000	-0.0000	0.0000	1.7040	0.1230
Pair 4	MA – AA	0.0000	0.0000	-0.0000	.00005	1.9670	0.0810
Sum SSIM	MA – AA	0.0950	0.1725	-0.0284	0.2184	1.7410	0.1160

CONCLUSIONS

This work has established a comparative analysis of additive and multiplicative noise based automated regularizations in nonlinear diffusion image processing based on four modified versions of the Perona-Malik model. The simulation results and paired samples T-tests revealed that the multiplicative noise based automation presents convincing results compared to the additive noise based automation for small speckle variances while performances of the two automation methods do not significantly differ for large speckle variances. It should be noted that the multiplicative noise based automation stay quasi stable after attaining the peak value and therefore, it is recommended for speckle reduction. The multiplicative and additive noise variances are assumed known in priori and that this is not the case in real environment. Therefore, to fully automate the regularization parameter, the noise variance should be estimated based on speckles distribution in the image. The estimation of speckle noise variances remains an open research problem for future studies.

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