THE DESIGN OF A RESONANT POST WAVEGUIDE FILTER

By

Ramjee Prasad and H.R. Mgombelo

ABSTRACT

A seven stage, quarter-wave coupled, single post, resonant cavity type waveguide filter has been studied. The posts are placed according to the Chebyscheff distribution. The filter is designed and fabricated to meet the following specifications: Centre frequency: 9.31 GHz, Number of stages = 7, 3-dB bandwidth = 300 Mhz, Ripple tolerance in the passband = 0.5 dB. The frequency behaviour of this filter has been experimentally studied and is in close agreement with the specifications.

1.0 INTRODUCTION

Within our Telecommunications Research Group, it has been planned to develop technical know-how for various microwave components which will enable us to manufacture them indigenously. In the series of development, we have selected microwave filters as our first task.

From the beginning of microwave engineering until today, filter design has been a steady and fruitful field for investigations. There are two very good reasons for the continuing attraction of microwave filters. The first is the over increasing importance of filters to microwave systems, as these systems become more complex and as the frequency spectrum becomes more densely filled with signals. The second is a remarkable appeal of microwave filters for creative study which results from the amenability of filter circuits to theoretical analysis and the close experimental agreements that can usually be achieved.

Microwave filters can be designed with either maximally flat or Chebyscheff response in the pass band.

1 Associate Professor, Department of Electrical Engineering, University of Dar es Salaam.

2 Senior Lecturer, Department of Electrical Engineering, University of Dar es Salaam.
This paper presents the design of microwave filters with Tchebyscheff or equal ripple response in the bass-band. The striking feature of the Tchebyscheff response curve compared with the maximally flat curve is its much greater rate of rise beyond cut-off point. This means that the corresponding filter has a much sharper cut-off region separating the pass-band and stop-band, which is usually a desired characteristic.

In general, the Tchebyscheff filter approximates the ideal "brick wall" characteristic outside the pass-band. The penalty, however, is the ripples within the pass band.

Waveguide filters can be constructed in two ways. In the first, waveguide cavities are formed by inductive arises or posts spaced approximately a half wavelength apart, the individual cavities being coupled to each other through quarter-wavelength waveguide transformers. In the other, adjacent cavities are directly coupled to each other by a single iris or post. The quarter-wavelength coupled filters permit tuning of the individual cavities before assembly. The alignment procedures and iris (Or post) design is complicated in direct coupled cavity filters even though it is capable of successful operation over a greater bandwidth. In addition to this, the resonators are easily tested individually.

For quarter-wavelength coupled cavities, the transmission characteristic is flatter in the passband, steeper in the rejection band; and when losses are considered, the attenuation is smaller at resonance and the VSWR is by far superior.

For this paper the design of seven-stage, Tchebyscheff, band-pass, rectangular waveguide, X-band filter is presented with the following specifications:

- Band edges at \( f_1 = 9.16 \, \text{GHz} \) and \( f_2 = 9.46 \, \text{GHz} \),
- Permissible ripple tolerances in the pass-band = 0.5dB.

With the above mentioned specifications a filter was fabricated and its theoretical and experimental characteristics were compared. It was experimentally found that the filter has 0.5 dB ripple tolerance in the passband with 300 MHz bandwidth. The results, therefore, exhibit satisfactory correlation between design and theory.
2.0 DESIGN THEORY

In this paper, waveguide filters (1-6) that are of the band-pass, employing inductive posts as the primary reactive element are considered. These filters are of the cavity type where two identical single posts (longitudinally spaced approximately a half wavelength along the guide) make up one stage of the filter. The tuning of a single post resonant cavity is accomplished by one screw symmetrically placed between the two identical posts of the cavity.

Microwave band-pass filters with Tchebyscheff response can be designed if the centre frequency, 3dB bandwidth and ripple tolerance are known. The steepness of the response curve beyond the cutoff depends on the number of stages, therefore, the number of stages, "n" is selected as per requirement.

After selecting the number of stages the normalised reactive low-pass filter proto-type parameters are determined from Weinberg's table (1) for the given ripple tolerance.

The loaded $Q_e$ of the filter is then calculated using the equation:

$$ Q_e = \frac{\omega_0}{\omega_1} $$

.........(1)

where $\omega_0$ is the centre frequency and $\omega_1$ is the 3dB bandwidth.

The bandwidth correlation factor for the given number of stages and ripple tolerance is calculated from the equation (1).

$$ 3dB = 10 \log \left( 1 + \left( 10^{Am/10} - 1 \right) \cosh^2 \left( n \cosh^{-1} a \right) \right) $$

.......................................................... (2)

where $Am$ is the ripple tolerance level in dB corresponding to the normalised $\omega = 1$ point,

$n$ is the number of resonators in the filter,

$\omega_c$ is the bandwidth correlation factor and

$\omega$ is the ripple tolerance bandwidth for Tchebyscheff design.

For each resonator, the loaded $Q_e, Q_m$ is calculated using the following expression:
\[ Q_{1e} = \frac{g_s}{2} Q_0 - m \frac{\lambda_{\mu}}{\lambda_0} \pi \frac{\lambda}{\lambda_0} \]

\[ \text{................................. (3)} \]

where \( g_s \) is the normalised reactive low-pass prototype parameter, 
\( a \) is the bandwidth correlation factor,
\( 1, \) for \( K = 1, \) and \( n \)
\( M = 2, \) for \( k = 2, 3, \ldots, n-1, \)
\( \lambda_{\mu} \) is the guide wavelength at the center frequency and
\( \lambda_s \) is the free space wavelength at center frequency.

Next the normalised inductive susceptance \( B \) is found for a given value of \( f/f_c \) from the following relation

\[ Q_i = \frac{1}{4} \left( \frac{\lambda g}{\lambda} \right)^2 \left( -B - \frac{B^2}{4 \tan^2 \frac{\lambda}{2} + \frac{2B^2}{B^2 + 4}} \right) \]

where \( \lambda_s \) is the guide wavelength,
\( \lambda \) is the free space wavelength
\( B \) is the normalised inductive susceptance and
\( Q_i \) is the loaded \( Q. \)

The post diameters are calculated using the following equations: \( (7) \)

\[ \frac{x_s}{Z_0} = \frac{x_s}{z_0} = \frac{a}{2 \lambda g} \left( S_0 - \frac{\pi d}{2 \lambda} \right)^2 - \frac{5}{8} \left( \frac{\pi d}{2 \lambda} \right)^2 - \frac{2}{3} \left( \frac{\pi d}{2 \lambda} \right)^4 \left( S_2 - 2 S_0 \frac{\lambda^2}{\lambda g^2} \right) \]

\[ \text{............................... (5)} \]

\[ \frac{x_s}{Z_0} = \frac{1}{1 + \frac{\pi d}{2 \lambda} \left( S_2 + \frac{\lambda^2}{\lambda g^2} \right)} \]

where,
\( S_0 = \ln \left( \frac{4a}{\pi d} \right) - 2 + 2 \sum_{s,t=0}^{\infty} \frac{1}{n^2 - \left( \frac{s+t}{n} \right)^2} - \frac{1}{n} \)

\( S_2 = \ln \frac{4a}{\pi d} \frac{5}{2} + \frac{11}{3} \left( \frac{\lambda}{2a} \right)^2 + \sum_{s,t=0}^{\infty} \sqrt{n^2 - \left( \frac{2a}{\lambda} \right)^2} - \frac{n}{n} + \frac{2}{\pi} \left( \frac{a}{\lambda} \right)^2 \)
\[ a = \text{height dimension of the waveguide} \]
\[ d = \text{diameter of the post} \]
\[ X_b = \text{capacitive reactance in the equivalent circuit} \]
\[ X_a = \text{inductive reactance} \]
\[ Z_0 = \text{characteristic impedance.} \]

Lastly the separation between the posts and length of the quarter-wave length couplings is found from the relation

\[ L_s = \frac{\lambda g_o}{2\pi} \left( \frac{1}{\pi - \tan^{-1}\left( \frac{2}{B} \right)} \right) \]

\[ S_s = L_s = \frac{L_s}{2} + \frac{\lambda g_o}{4} \]

Where, \( L_s \) = length dimensions between posts with \( K = 1, 2, \ldots, n \) for the cavities.

\( S_s = 1, k = \text{length of the quarter wave couplings with } k = 2, 3, \ldots, n \)

### 3.0 DESIGN PROBLEM

The design of a microwave, band pass filter with the following specifications is presented here:

- **Centre frequency** = 9.31 GHz
- **3dB bandwidth** = 300 MHz
- **Ripple tolerance** = 0.5 dB
- **No. of stages** = 7
- **Types of distribution** = Tchebysheff.

**Design Steps**

1. The normalised reactive low pass filter prototype parameters are obtained

\[ g_1 = g_7 = 1.7372 \]
\[ g_2 = g_6 = 1.2583 \]
\[ g_3 = g_5 = 2.6381 \]
\[ g_4 = 1.3444 \]

2. The loaded Q of the filter from equation (1)

\[ Q_0 = 31.033 \]

3. The bandwidth correlation factor \( a \), for a 0.5 dB ripple tolerance from equation (2)

\[ a = 1.029 \]

4. The center guide wavelength is determined from the relation:

\[ \lambda_{cg} = \frac{1}{2}(\lambda_{g1} + \lambda_{g2}) \]

Where \( \lambda_{g1} \) is the guide wavelength at \( f_1 = 9.16 \) GHz and \( \lambda_{g2} \) is the guide wavelength at \( f_2 = 9.46 \) GHz cut off frequency (fc) for an X-band waveguide (0.9" x 0.5") is 6.56 GHz

\[ \lambda_{g1} = 1.847 \text{ inches} \]
\[ \lambda_{g2} = 1.733 \text{ inches} \]
\[ \lambda_{fc} = 1.790 \text{ inches} \]
\[ \lambda_c = 1.270 \text{ inches} \]

5. The loaded \( Q \), \( Q_{lx} \) of each resonator can be calculated using equation (3)

\[ Q_{l1} = 26.93 = Q_{l7} \]
\[ Q_{l2} = 18.52 = Q_{l6} \]
\[ Q_{l3} = 40.62 = Q_{l5} \]
\[ Q_{l4} = 21.46 \]

6. The ratio of center frequency to waveguide cutoff frequency is

\[ \frac{f}{f_c} = 1.419 \]

7. The normalised inductive susceptance from equation (4) are:

\[ B_1 = B_7 = 40 \]
\[ B_2 = B_6 = 3.3 \]
\[ B_3 = B_5 = 4.9 \]
\[ B_4 = 3.39 \]

8. The \(|\cdot|\) ratio of each post (from equation 5) are:

\[ \frac{d_1}{a} = 0.07 \]
\[ \frac{d_2}{a} = 0.057 \]
\[ \frac{d_3}{a} = 0.09 \]
\[ \frac{d_4}{a} = 0.06 \]

Since \(a = 0.9\) inches, the diameters of the posts are:

\[ d_1 = d_1 = 0.063\] inches
\[ d_2 = d_4 = 0.0513\] inches
\[ d_3 = d_3 = 0.081\] inches
\[ d_4 = 0.054\] inch.

9. The separation of the posts is obtained from equation (6):

\[ L_1 = L_2 = 0.7615\] inch.
\[ L_2 = L_6 = 0.7398\] inch.
\[ L_3 = L_5 = 0.7848\] inch.
\[ L_4 = 0.7430\] inch.

and \(S_{1,2} = S_{6,7} = 0.30315\) inch.
\[ S_{2,3} = S_{5,6} = 0.3145\] inch.
\[ S_{3,4} = S_{4,5} = 0.3161\] inch.

The design filter is shown in Fig.1

4.0 INSERTION LOSS

The power loss ratio for the equal ripple or Tchebyscheff low-pass filter is given by (2)
\[
P_{18} = 1 + K^2 T_2^2 \left( \frac{\omega_c}{\omega} \right)
\]

where

\[
-T_2 \left( \frac{\omega}{\omega_c} \right) = \cosh \left( \frac{n}{\cosh^{-1} \left( \frac{\omega}{\omega_c} \right)} \right)
\]

\[k = 10^{A_m/10} - 1\]

\(\omega_c\) is the cut off frequency of the filter and

- \(A_m\) is the pass band ripple in dB.

Equation (7) can also be expressed as

\[
P_{18} = 1 + \left( 10^{A_m/10} - 1 \right) \cosh^2 \left( \frac{n}{\cosh^{-1} \left( \frac{\omega}{\omega_c} \right)} \right)
\]

A bandpass filter from the low-pass filter is obtained using the following frequency transformation:

\[
\omega = \frac{\omega_s}{\omega_c} - \omega_c \left( \frac{\omega}{\omega_c} - \omega_s \right)
\]

where \(\omega\) corresponds to any frequency

\(\omega_c\) corresponds to the center frequency and

\([\omega_c - \omega_c]\) to the bandwidth of the filter.

The expression of the power loss ratio of a bandpass filter, using, equations (8) and (9) becomes:

\[
P_{18} = 1 + \left( 10^{A_m/10} - 1 \right) \cosh^2 \left( \frac{n}{\cosh^{-1} \left( \frac{\omega}{\omega_c} \right)} \right)
\]

Thus the insertion loss of a Tchebyscheff filter in dB is expressed as
\[ L = 10 \log (Pha) \]
\[ = 10 \log \left( 1 + \left[ 10^{4m/10} - 1 \right] \cosh^2 \left( \frac{n \cosh^{-1} \omega}{2} \right) \right) \]

\[ \text{For } n = 7 \text{ and } Am = 0.5 \text{ dB the corresponding insertion loss becomes:} \]
\[ L = 10 \log \left( 1 + 0.122 \left( 64 \omega^7 - 112 \omega^5 + 56 \omega^3 - 7 \omega \right) \right)^2 \]

\[ \text{For various values of } \omega, L \text{ is calculated using equation (12). Theoretical calculations show that a minimum insertion loss of zero dB occurs at the center frequency, i.e. } \omega = 0 \text{ and } \omega = 1.0. \text{ The maximum values of insertion loss in the pass band occur at } \omega = 0.24, 0.62, 0.9. \text{ In the stop band the insertion loss increases steeply beyond } \omega = 1.0 \text{ point.} \]

5.0 EXPERIMENTAL RESULTS

Once the filter has been designed and fabricated, it is desirable to insure that insertion loss, bandwidth and \( Q \) are within the required limits. The filter is tuned by adjusting all the resonators to resonate at the midband frequency.

Properly tuned band pass filters will have a response that is symmetrical about the midband frequency.

Using the standard technique (1), the fabricated filter was properly tuned. After tuning the filter, the VSWR of the filter was measured in the frequency range from 3.09 GHz to 3.53 GHz. The insertion loss was calculated using the expression.

\[ L = 10 \log \frac{1}{1 - \rho^2} \]

where \( L = \) is the insertion loss in dB
\( \rho \) is the reflection coefficient.
\[ \rho = \frac{VSWR - 1}{VSWR + 1} \]

where VSWR is the voltage standing wave ratio

Fig. 2 shows the measure (dashed curve) and theoretical (solid curve) (calculated from eq. 12) insertion loss characteristics of the seven stage, quarter wave, coupled, single post, resonant cavity type waveguide filter with a ripple tolerance of 0.5 dB in the passband.

The minimum value of the theoretical insertion loss is zero dB whereas the experimental value of minimum insertion loss is 0.48 dB. Practically it is impossible to obtain an insertion loss of zero dB as shown in the theoretical plot. Generally the deviation of the experimental value from zero dB is small and usually it is found to be less than 0.5 dB. The filter is designed to have 0.5 dB ripple in the pass band and this has been exhibited by the experimental insertion loss characteristics. The bandwidth of the filter obtained experimentally is 300 MHz which is the same as specified by the design.

The value of the total Q obtained experimentally is 30.99 compared to the design values of 31.033.

6.0 CONCLUSION

In this paper the design procedure for a quarter-wave, coupled, bandpass filter has been discussed. A step-by-step method has been shown by illustrating the design of a seven stage, quarterwave, coupled, bandpass filter with center frequency 9.31 GHz, 300 MHz bandwidth and 0.5 dB ripple tolerance.

It has been emphasized that in order to ensure that the filter delivers maximum power at the output terminals, with minimum insertion loss, proper tuning is required. For proper tuning, the screws must be perpendicular to the broadwall of the waveguide, otherwise the filter performance deteriorates from the theoretical calculations.

The design procedure mentioned here can be used satisfactorily to design a waveguide filter with loaded Q's ranging from 4 to 200.

The frequency behaviour of the filter has been experimentally studied and is in close agreement with the specifications.
It is worth to mention here that the workshop facility available in the Faculty of Engineering, University of Dar es Salaam is quite appropriate for fabricating waveguide filters and the Telecommunication Laboratory has adequate facility for testing the microwave filters for X-band.
7.0 REFERENCES


Fig. 1: Designed seven stage wave guide filter.
Fig. 2 Measured (dashed curve) and theoretical (solid curve) INSERTION LOSS CHARACTERISTICS of the seven stage filter