OPTIMAL ALLOCATION OF WATER AMONG INDUSTRIES IN DAR ES SALAAM CITY, TANZANIA

by

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ABSTRACT

Dar es Salaam is the city with the largest number of industries in Tanzania. However, the water demand for these industries is more than the supply. Therefore, there is a water use competing problem among the industries. Dynamic programming technique has been applied to optimally allocate the available water among the industries with the objective of maximizing the net benefits. Results reveal that, industries with the highest net return per unit of water will get the first priority of being allocated water.

1. INTRODUCTION

The old cities grew up in the valleys of major rivers. Water plays an important role in the growth of cities, agricultural activities, manufacturing industries etc. The main uses of water to man are : domestic, agricultural, industrial, commercial, recreation, power generation and navigation. There are conflicting and competing uses of water. Competing uses of water is the problem which is being addresed in this paper with reference to the city of Dar es Salaam.

Dar es Salaam city lies approximately at latitude 6.8 South and longitude 39.3 East. Dar es Salaam is the largest city in Tanzania with an estimated population of about 1.4 million people (JICA, 1984). The water demand is presently estimated to be 260,000 cubic meters per day. The demand for water will always increase as more industries are constructed and more people migrate into the city. Currently the water supply is estimated to be 115,000 cubic meters per day. From the water supply and demand figures, it can be seen that, the supply does not meet the demand. Therefore many parts of the city get less water than they require and some parts do not get water at all, thus causing under production or total closing down of some industries.

It is assumed in this study that the domestic water demand is met and the problem that remains is that of optimal allocation of water among competing industries. Thus the major objective of this study is to develop a water allocation policy that will optimally allocate whatever available quantity of water among the industries in such a way as to maximize the net benefits.

Eleven industries have been considered in this study. These industries are: Tanzania Distilleries (TDL); Coastal Dairy (CDL); ASBESCO (ASB); Tanganyika Parkers (TPL); Dar Brew Kibuku (DBK); KILTEX (.KTM); Kibo

Paper Mill (KPM); Tanzania Breweries (TBL); Tanzania Cigarate Company (TCC); Friendship Textile Mill (FTM); and Tanganyika Dyeing and Weaving (TDW).

There are many optimization techniques which have been used to solve resource allocation problems. Linear, nonlinear, posynomial, geometric and dynamic programming and combinations of these techniques have been successfully used to manage water resource systems (Lasdon, 1970; Hall and Dracup, 1970; Reddy and Clyma, 1982; Murray and Yakowitz, 1979; Young, 1967). Dynamic programming has been used extensively to allocate scarce resources in multistage decision process. Dynamic programming techniques are based on the "Principle of Optimality" (Bellman, 1957). Dynamic programming technique will be used in this study.

2. SOLUTION CONFIGURATION

The standard form of dynamic programming (DP) is expressed as :

Min or Max

$$\sum_{t=1}^{N} B_t(X_t).... \tag{1}$$

Subject to

$$\sum_{t=1}^{N} X_t \le X , t = 1,2,,N$$

Where $B_t(X_t)$ is the return function for allocating X units of water to stage t or industry t and X is the total amount of water available for allocation to all industries, and N is the total number of industries. The general backward DP recursive equation is expressed as :

$$F_t(X_t) = \text{Min or Max}\, B_t(U_t) + F_{t-1}(X_{t-1}) \tag{2}$$

Subject to

$$U_t = X - X_t$$

$$0 \le U_t \le X$$

$$X_{Min} \le X_t \le X_{Max}$$

The water use benefit function $B_t\ (U)_t$ for all industries have been developed by Macha (1987). The water use benefit functions are presented in Table 1.

Table 1: Water Use Benefit Functions in Million Shillings (Macha, 1987)

(X) $10^4 m^3$	(TCC) $B_1(U_1)$	(ASB) $B_2(U_2)$	(TDL) $B_3(U_3)$	(CDL) $B_4(U_4)$	(TPL) $B_5(U_5)$	(DBK) $B_6(U_6)$	(KTM) $B_7(U_7)$	(KPM) $B_8(U_8)$	(TDW) $B_9(U_9)$	(TBL) $B_{10}(U_{10})$
0	-7.6	-0.04	-0.2	-0.2	-16.8	-0.04	-0.05	0	-0.05	-1.1
5	31.21	1.47	3.37	1.79	95.55	0.82	0.28	o.03	0.37	1.85
10		2.97	6.93	3.73	207.6	1.68	0.6	0.05	0.78	4.79
15						2.54	0.92	0.08	1.2	7.74
20							1.24	0.11	1.61	10.68
25							1.56	0.14	2.03	13.63
30								0.16	2.44	16.57
35										19.52
40										22.46
45										
50										
55										
60										
65										
70										
75										
80										

The industries are conceptualized as stages. Therefore, for the last stage problem, the DP optimal return function corresponds to the water use benefit function (i.e., column 11 Table 1). For the 10th stage problem, which is the 10th industry, the solution is as follows:

STAGE 10

X_{10}	X_{11}	$X_{10}-X_{11}$	$B_{10}(U_{10})$	$F_{11}(X_{11})$	$F_{10}(X_{10})$	$X_{11}(X_{10})$	$U_{10}(X_{10})$
0	0	0	-1.1	-0.06	-1.1	0	0
5	0	5	1.85	-0.06	1.79	5	5
	5	0	-1.1	0.06			
10	0	10	4.7	-0.06	4.73		
	5	5	1.85	-0.06			
	10	0	-1.1	0.17			
15	0	15	7.74	-0.06	7.68	0	15
	5	10	4.79	0.06			
	10	5	1.85	0.17			
	15	0	-1.1	0.29			
20	0	20	10.68	-0.06	10.62	0	20
	5	15	7.74	0.06			
	10	10	4.79	0.17			
	15	5	1.85	0.29			
	20	0	-1.1	0.4			
25	0	25	13.63	-0.06	13.57	0	25
	5	20	10.68	0.06			
	10	15	7.74	0.17			
	15	10	4.79	0.29			
	20	5	1.85	0.4			
	25	0	-1.1	0.25			

STAGE 9

X_q	X_{10}	$X_9 - X_{10}$	$B_9(U_9)$	$F_{10}(X_{10})$	$F_9(X_9)$	$X_{10}(X_9)$	$U_9(X_9)$
0	0	0	0	-1.66	-1.66	0	0
5	0	5	0.03	-1.66			
	5	0	0	2.29	1.29	5	0
10	0	10	0.05	-1.66			
	5	5	0.03	1.29			
	10	0	0	4.23	4.23	10	0
15	0	15	0.08	-1.66			
	5	10	0.05	1.29			
	10	5	0.03	4.23			
	15	0	0	7.1	7.18	15	0
20	0	20	0.11	-1.66			
	5	15	0.08	1.29			
	10	10	0.05	4.23			
	15	5	0.03	7.18			
	20	0	0	10.12	10.12	20	0
25	0	25	0.14	-1.66			
	5	20	0.11	1.29			
	10	15	0.08	4.23			
	15	10	0.05	7.18			
	20	5	0.03	10.12			
	25	0	0	13.07	13.07	25	0
30	0	30	0.16	-1.66			
	5	25	0.14	1.29			
	10	20	0.11	4.23			
	15	15	0.08	7.78			
	20	10	0.05	10.12			
	25	5	0.03	13.07			
	30	0	0	15.01	15.01	30	0

STAGE 8

X_8	X_9	X_8-X_9	$B_8(U_8)$	$F_9(X_9)$	$F_8(X_8)$	$X_9(X_8)$	$U_8(X_8)$
0	0	0	-0.5	-1.16	-1.16	0	0
5	0	5	0.37	-1.16			
	5	0	-0.5	1.79	1.29		
10	0	10	0.78	-1.16			
	5	5	0.37	1.79			
	10	0	-0.5	4.73			
15	0	15	1.2	-1.16			
	5	10	0.78	1.79			
	10	5	0.37	4.73			
	15	0	-0.5	7.68	7.18	15	0
20	0	20	1.61	-1.16			
	5	15	1.2	1.79			
	10	10	0.78	4.73			
	15	5	0.37	7.68			
	20	0	-0.5	10.62	10.12	20	0
25	0	25	2.03	-1.16			
	5	20	1.61	1.79			
	10	15	1.2	4.73			
	15	10	0.78	7.68			
	20	5	0.35	10.62			
	25	0	-0.5	13.57	13.07	25	0
30	0	30	2.44	-1.16			
	5	25	2.03	1.79			
	10	20	1.61	4.73			
	15	15	1.2	7.68			
	20	10	0.78	10.62			
	25	5	0.37	13.57			
	30	0	-0.5	15.51	15.01	30	0

STAGE 7

X_7	X_8	X_7-X_8	$B_7(U_7)$	$F_8(X_8)$	$F_7(X_7)$	$X_8(X_7)$	$U_7(X_7)$
0	0	5	-0.05	-1.66	1.71	0	0
5	0	0	0.28	-1.66			
	5	10	-0.05	1.29	1.24	5	0
10	0	0	0.6	-1.66			
	5	5	0.28	1.28			
	10	10	-0.05	4.23	4.18	10	0
15	0	15	0.92	-1.66			
	5	0	0.6	1.29			
	10	5	0.28	4.23			
	15	20	-0.05	7.18	7.13	15	0
20	0	10	2.24	-1.66			
	5	15	0.92	1.29			
	10	0	0.6	4.23			
	15	5	0.28	7.18			
	20	20	-0.05	10.12	10.07	20	0
25	0	25	1.56	-1.66			
	5	10	1.24	1.29			
	10	15	0.92	4.23			
	15	0	0.6	7.18			
	20	5	0.28	10.12			
	25	0	-0.05	13.07	13.02	25	0

STAGE 6

X_6	X_7	X_6-X_7	$B_6(U_6)$	$F_7(X_7)$	$F_6(X_6)$	$X_7(X_6)$	$U_6(X_6)$
0	0	0	-0.04	-1.71	-1.75	0	0
5	0	5	0.82	1.71			
	5	0	-0.04	1.24	1.2	5	0
10	0	10	1.68	-1.71			
	5	5	0.82	1.24			
	10	0	-0.04	4.18	4.14	10	0
15	0	15	2.54	-1.71			
	5	10	1.68	1.24			
	10	5	0.82	4.18			
	15	0	-0.04	7.13	7.09	15	0

STAGE 5

X_5	X_6	X_5-X_6	$B_5(U_5)$	$F_6(X_6)$	$F_5(X_5)$	$X_6(X_5)$	$U_5(X_5)$
0							
5	0	0	-16.8	-1.75	-18.55	0	0
10	0	5	95.55	-1.75	93.8	0	5
	5	0	-16.8	1.2			
	0	10	207.6	-1.75	205.85	0	10
	5	5	95.5	1.2			
	10	0	-16.8	4.14			

STAGE 4

X_4	X_5	X_4-X_5	$B_4(U_4)$	$F_5(X_5)$	$F_4(X_4)$	$X_5(X_4)$	$U_4(X_4)$
0	0	0	-0.2	-18.55	-18.75	0	0
5	0	5	1.79	-18.55			
10	5	0	-0.2	93.8	93.6	5	0
	0	10	3.78	-18.55			
	5	5	1.79	93.8			
	10	0	-0.2	205.85	205.65	10	0

STAGE 3

X_3	X_4	X_4-X_5	$B_3(U_3)$	$F_4(X_3)$	$F_3(X_3)$	$X_4(X_3)$	$U_3(X_3)$
0	0	0	-0.2	-18.75	-18.75	0	0
5	0	5	3.37	-18.75			
	5	0	-0.2	93.6	93.4	5	0
10	0	10	6.93	-18.75			
	5	5	3.37	93.6			
	10	0	-0.2	205.65	205.45	10	0

STAGE 2

X_2	X_3	X_3-X_2	$B_2(U_2)$	$F_3(X_3)$	$F_2(X_2)$	$X_3(X_2)$	$U_2(X_2)$
0	0	0	-0.04	-18.95	-18.99	0	0
5	0	5	1.47	-18.95			
	5	0	-0.04	93.4	93.36	5	0
10	0	10	2.97	-18.95			
	10	0	-0.04	205.45	205.41	10	0

STAGE 1

X_1	X_2	X_1X_2	$B_1(U_1)$	$F_2(X_2)$	$F_1(X_1)$	$X_2(X_1)$	$U_1(X_1)$
0	0	0	-7.6	-18.99	-26.59	0	0
	0	5	31.21	-18.99			
5	5	0	-7.6	93.36	85.76	5	0
	0	10	31.21	-18.99			
10	5	5	31.21	93.6			
	10	0	-7.6	205.41	197.81	10	0

The obtained results can now be used to trace-back the optimal quantity of water to allocate to each industry given a certain quantity of water, in order to maximize the net benefits. Suppose we have 50 x 10^4m^3 (i.e., X = 50 x 10^4m^3) of water and it is desired to optimally allocate the water amongthe industries, the trace-back procedure is as follows:

Looking at the obtained results and starting from stage one (i.e., TCC) and starting with 50 x 10^4m^3 of water, the optimal quantity of water to allocate to TCC is $U_1(X_1)$ = 0. Moving to stage 2 (i.e., ASBESCO is $U_2(X_2)$ = O. Moving to stage 3 (i.e., TDL) is $U_3(X_3)$ = 0. Continue doing the same till your each stage 11 (i.e., FTM). The optimal water allocation strategy is given in summary below:

$U_i(X_i)$ = 0 for i = 1,2,3,4,6,7,8,9,11
$U_5(X_5)$ = 10 x 10^4m^3 and $U_{10}(X_{10})$ = 40 x 10^4m^3

The optimal return value or net benefits for allocation 50 x 10^4m^3 to eleven industries is 221.37Mshs. If X = 60 x 10^4m^3 of water, the optimal water allocation strategy is as follows:

$U_1(X_1)$ = 5 x 10^4m^3; $U_5(X_5)$ = 10 x 10^4m^3
$U_3(X_3)$ = 10 x 10^4m^3; $U_{10}(X_{10})$ = 35 x 10^4m^3

The optimal water allocation to remaining industries is zero. Theoptimal return value for allocating 60 x 10^4m^3 to eleven industries is 267.11Mshs.

Suppose due to political reasons each industry has to receive a certain quantity of water i.e. $U_t(X_t) > 0$ for all t. Then for X = 60 x 10^4m^3 of water the optimal allocation strategy is as follows:

$U_t(X_t)$ = 5 x 10^4m^3 for t = 1,2,3,.........,11 except
$U_5(X_5)$ = 10 x 10^4m^3.

The optimal return value is 248.85Mshs.

SUMMARY AND CONCLUSIONS

The city of Dar es Salaam Tanzania gets a finite quantity of water. Dynamic programming technique has been applied to optimally allocate whatever available quantity of water among eleven industries. Hand calculations have been performed in this study but a personal computer could be handy. Since we are living in the erra of computers, water managers should use such techniques to optimally manage their water systems.

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